

FORMATION OF GALACTIC NUCLEI

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Draft version December 20, 2001

ABSTRACT

We investigate a model in which galactic nuclei form via the coalescence of pre-existing stellar systems containing supermassive black holes. Merger simulations are carried out using N -body algorithms that can follow the formation and decay of a black-hole binary and its effect on the surrounding stars down to sub-parsec scales. Our initial stellar systems have steep central density cusps similar to those in low-luminosity elliptical galaxies. Immediately following the merger, the density profile of the remnant is homologous with the initial density profile and the steep nuclear cusp is preserved. However the formation of a black-hole binary transfers energy to the stars and lowers the central density; continued decay of the binary creates a $\rho \sim r^{-1}$ density cusp similar to those observed in bright elliptical galaxies, with a break radius that extends well beyond the sphere of gravitational influence of the black holes. Our simulations are the first to successfully produce shallow power-law cusps from mergers of galaxies with steep cusps, and our results support a picture in which the observed dependence of nuclear cusp slope on galaxy luminosity is a consequence of galaxy interactions. We discuss the implications of our results for the survivability of dark-matter cusps.

We follow the decay of the black hole binary over a factor of ~ 20 in separation after formation of a hard binary, considerably farther than in previous simulations. We see almost no dependence of the binary's decay rate on number of particles in the simulation, contrary to earlier studies in which a lower initial density of stars led to a more rapid depletion of the binary's loss cone. We nevertheless argue that the decay of a black hole binary in a real galaxy would be expected to stall at separations of $0.01 - 1$ pc unless some additional mechanism is able to extract energy from the binary.

1. INTRODUCTION

Galactic nuclei¹ are regions of high stellar density at the centers of galaxies. Early studies of the evolution of galactic nuclei (Spitzer & Saslaw, 1966; Spitzer & Stone, 1967; CoGata, 1967; Sanders, 1970) emphasized stellar encounters and collisions as the dominant physical processes. In these models, the density of a compact stellar system gradually increases as energetic stars are scattered into elongated orbits via two-body encounters. The increase in density leads to a higher rate of physical collisions between stars, liberating gas that falls to the center of the system and condenses into new stars which undergo further collisions. Beggelman & Rees (1978) argued that the evolution of a dense nucleus would lead inevitably to the formation of a massive black hole (BH) at the center, either by runaway stellar mergers or by creation of a massive gas cloud which collapses. Subsequent studies (Duncan & Shapiro, 1983; Quinlan & Shapiro, 1987, 1989; David, Durisen & Cohn, 1987a,b; Murphy, Cohn & Durisen, 1991) have included “seed” BHs which grow via accretion of stars or gas liberated by stellar collisions.

The fundamental time scale in these models is the relaxation time determined by the stars, or

$$t_r \approx 0.34 \frac{\sigma_*^3}{G^2 m_* \rho \ln \Lambda} \quad (1)$$

(Spitzer & Hart, 1971), where σ_* is the stellar 1D velocity dispersion, m_* and ρ are the stellar mass and mass density, and $\ln \Lambda$ is the Coulomb logarithm. Observations before

¹We use the term “nucleus” to refer generically to the central parts of galaxies. The term is sometimes used more restrictively to refer to pointlike nuclei, e.g. Kormendy & McClure (1993).

about 1990 lacked the resolution to determine whether t_r was shorter than a Hubble time on scales smaller than ~ 1 pc in galactic nuclei. We now know that stellar densities increase approximately as power laws in galactic nuclei, $\rho \sim r^{-\gamma}$, down to the smallest radii that can be resolved, or roughly 10^{-1} pc in the nearest galaxies. For instance, the Local Group galaxies M31, M32 and the Milky Way all have nuclear density cusps with $\gamma = 1.5 \pm 0.5$ (Lauer et al., 1998). Furthermore the evidence for supermassive BHs is compelling in these galaxies. Within the BH's sphere of influence r_{gr} , where

$$r_{gr} \equiv \frac{GM_\bullet}{\sigma_*^2} \approx 10.8 \text{ pc} \left(\frac{M_\bullet}{10^8 M_\odot} \right) \left(\frac{\sigma_*}{200 \text{ km s}^{-1}} \right)^{-2}, \quad (2)$$

the stellar velocity dispersion rises as $\sim r^{-1/2}$. Equation (1) then implies $t_r \propto r^{-3/2} \approx r^0$, nearly independent of radius. The relaxation time based on observations at $\sim 0.1''$ exceeds 10^{11} yr in almost all galaxies (Faber et al., 1997); this angular size corresponds roughly to a radius r_{gr} for nearby galaxies (Merritt & Ferrarese, 2001b), from which it follows that t_r is likely to exceed a Hubble time at smaller radii as well. (The pointlike nucleus of M33 is probably an exception; see Kormendy & McClure (1993).)

Physical collisions between stars occur on a timescale that is longer than t_r by roughly a factor $(\ln \Lambda) \Theta^2 / (1 + \Theta) \approx 10$ where Θ , the “Safronov number” (Safronov, 1966), is of order a few for stars in a galactic nucleus. Thus neither elastic nor inelastic gravitational encounters are likely to be of dominant importance in determining the structure of nuclei containing supermassive BHs.

Nevertheless the properties of galactic nuclei do vary in systematic ways with the properties of their host galaxies (Kormendy, 1985; Lauer, 1985; Faber et al., 1997) and one

would like to understand this. Recent discussions of the formation and evolution of galactic nuclei have begun from the assumption that supermassive BHs were created during the quasar epoch and have been present ever since with roughly their current masses. Another element missing from the earlier studies was galactic mergers. Mergers are complex phenomena, but an almost certain consequence of a merger is the infall of the progenitor galaxy's BHs into the nucleus of the merged system (Begelman, Blandford & Rees, 1980). An infalling BH would be expected to carry with it a mass in stars of order its own mass, and decay of the BHs' orbits would inject a substantial amount of energy into the stars, enough to determine the structure of the remnant nucleus out to a radius of several times r_{gr} (Ebisuzaki, Makino & Okumura, 1991). In this picture, the structure and kinematics of galactic nuclei are fossil relics of the merger histories of galaxies and of the interaction between stars and supermassive binary BHs.

The present paper is a numerical study of this formation model. We simulate the merger of two galaxies, each of which contains a central point mass representing a supermassive BH. Our study is unique in that it follows the details of the merger from its earliest stages, when the two galaxies are distinct, to its late stages, when the BHs have formed a hard binary and the binary has decayed via energy exchange with stars to a separation much less than one parsec. No existing N -body code can efficiently follow the evolution over such a wide range of scales; hence we break the calculation into two parts, before and after formation of the BH binary, and use different algorithms for each (§2).

We also include for the first time initial models which are self-consistent realizations of galaxies with steep central density cusps, $\rho \sim r^{-2}$. This choice is motivated by a number of lines of argument which suggest that the density of stars around a supermassive BH should be a steep power law. Random gravitational encounters between stars lead, over two-body relaxation times, to density profiles of the form $\rho \sim r^{-2.23}$ in the absence of a black hole (Cohn, 1980) and $\rho \sim r^{-1.75}$ in the presence of a BH (Bahcall & Wolf, 1976). It was argued above that relaxation processes are probably of secondary importance in most nuclei, but even the slow growth of a BH in a pre-existing, collisionless nucleus produces a density profile of the form $\rho \sim r^{-\gamma}$, $r \lesssim r_{gr}$ with $1.5 \lesssim \gamma \lesssim 2.5$ (Peebles, 1972; Young, 1980; Quinlan, Hernquist & Sigurdsson, 1995; van der Marel, 1999). Low-luminosity elliptical galaxies and the bulges of spiral galaxies are observed to have steep cusps, $1.5 \lesssim \gamma \lesssim 2.5$ (Ferrarese et al., 1994; Lauer et al., 1995); these galaxies are the least likely to have been strongly affected by mergers and hence their density profiles may reflect the structure of all nuclei at early times. Finally, hierarchical growth of structure in the universe generically produces systems with steep central density cusps, $\rho \sim r^{-1.5}$ (Dubinski & Carlberg, 1991; Navarro, Frenk & White, 1996; Moore et al., 1998), although simulations do not yet have sufficient resolution to make predictions on parsec or sub-parsec scales (Moore, 2001).

A major success of our study is the demonstration (§3.4) that the merger of two galaxies with steep, power-law density cusps can produce a galaxy with a shallow power-law cusp. Shallow cusps (also called “cuspy cores”) are ob-

served in the brightest elliptical galaxies (Merritt & Fridman, 1996; Gebhardt et al., 1996), and, while their origin has tentatively been associated with the binary BH model (Ebisuzaki, Makino & Okumura, 1991; Faber et al., 1997), no previous simulation had the resolution necessary to follow the coalescence of initially steep cusps. Our results support a model in which the observed dependence of nuclear cusp slope on galaxy luminosity is a consequence of galaxy interactions (§6).

We also discuss in detail (§4) the decay of the BH binary; we follow that decay over a factor ~ 20 in semimajor axis after formation of a bound pair, considerably farther than in earlier simulations. An important question is the dependence of the binary hardening rate on N , the number of particles in the simulation. Earlier studies had noted a decreasing decay rate with increasing N , implying that the decay in real galaxies, where N is very large, might be slow. We do not observe an appreciable N dependence in our simulations; we discuss the likely reasons for this in §4, but argue nevertheless that the decay would probably stall in real galaxies, at separations of $0.01 - 1$ pc, unless some additional mechanism is effective at extracting energy from the binary. We are therefore led to predict (§6) that some galaxies contain uncoalesced BH binaries at the current epoch.

We also present the morphological and kinematical structure of the merged galaxy on sub-parsec scales (§5) and discuss some observational signatures associated with our formation picture.

2. METHOD

In this section we describe the initial conditions and algorithms used in our simulations and compare them with those of other authors.

Initial conditions consisted of twin, spherical stellar systems following the density law

$$\rho(r) = \frac{M}{4\pi r_0^3} \left(\frac{r}{r_0} \right)^{-2} \left(1 + \frac{r}{r_0} \right)^{-2} \quad (3)$$

(Jaffe, 1983; Dehnen, 1993), where r_0 is the half-mass radius and M the total mass. To each of the models was added a central point of mass $M_\bullet = 0.01M$ representing the supermassive BH. The ratio M_\bullet/M in our models is somewhat greater than the mean ratio of BH mass to galaxy mass in observed galaxies, $\sim 1.2 \times 10^{-3}$ (Merritt & Ferrarese, 2001b); however the radius of influence of our BHs in our merged galaxies, $r_{gr} = GM_\bullet/\sigma_*^2 \approx 0.02$, is much smaller than r_0 which allows us to ignore the large-scale stellar distribution when scaling our models to the nuclei of real galaxies; see §6. Velocities of the stars were generated from the unique, isotropic distribution function (Tremaine et al., 1994) that reproduces the density law (3) in the combined potential of the stars and the central point mass. Thus our models are initially in a state of detailed equilibrium. The values chosen for the model parameters are listed in Table 1; Newton's constant is set to unity.¹ The galaxies were set at time $t = 0$ in an elliptic mutual orbit of semimajor axis $a_G = 2$ and velocity $v_G = 0.1425$ equal to half of the circular orbit velocity.

¹These parameters agree with the Heggie & Mathieu (1986) “standard” units.

TABLE 1
MODEL PARAMETERS

Parameter	Symbol	Value
Mass of Galaxy	M	1
Mass of Black Hole	M_\bullet	0.01
Half-Mass Radius	r_0	1
Total Energy	E	-0.25
Number of Stars	N	131,072
Stellar Mass	m_*	7.63×10^{-6}

(The index “G” labels the binary composed of two galaxies as distinct from a binary composed of two BHs.) Since the relative orbit rapidly circularizes, we do not expect our results to be strongly affected by the choice of orbital initial conditions.

Our goal was to follow the evolution of this binary system from its earliest stages, when the galaxies were distinct, through the formation of a bound BH pair, until the gradual exchange of energy between BHs and stars had caused the BH binary to shrink to sub-parsec separations. No single N -body code currently in existence can deal efficiently and accurately with evolution over such a broad range of scales; particularly demanding is the treatment of BH-BH and BH-star interactions, which require an algorithm that can accurately integrate the equations of motion of point masses without softening. The closest approximation to such a code is SCFBDY developed by G. Quinlan at Rutgers University and used in two published studies (Quinlan & Hernquist, 1997; Merritt & Quinlan, 1998). SCFBDY combined elements of Aarseth’s NBODY series of codes, including regularization of the BH-BH interaction, with a low-resolution, mean-field potential solver for computing the force field due to the stars. However SCFBDY is only suited to systems with a single density center and a high degree of symmetry, ruling out its application to mergers. SCFBDY also uses softened gravity for computing the BH-star interactions, an approximation that affects the accuracy of the critical interactions leading to the decay of the BH binary. This code is also not available in a form that runs on parallel architectures, limiting the number of particles that can be used.

We chose to break the problem into two parts, using a tree code for the early stages of the merger (roughly until the formation of a BH binary), and a high-precision, direct-summation code for the later stages. Ideally, one would use a high-precision code right from the start, to handle the steep force gradients produced by the BHs and the other stars in the cusps. However we were willing to accept some inaccuracy in the integrations during the early stages of the merger if in so doing we could treat a larger N ; our only prerequisite was that motions of stars on scales larger than the separation of the hard BH binary should be faithfully tracked during the early stages of the merger. This reasoning motivated our choice of the recently-released tree code GADGET (Springel, Yoshida & White, 2001) as the integrator for the early stages of the merger. Features of GADGET that are relevant here include domain decomposition of the particle data set, mapping of particles onto the classic octal tree structure that

respects hierarchical clustering of particles, quadrupolar expansion of force moments for spatially separated nodes, and individual and adaptive time steps for all particles. Force integration is controlled through the parameters (h, η) that denote, respectively, the gravitational softening length and the time step accuracy factor, $\Delta t = \eta/|\mathbf{a}|$. Special care was taken to identify parameter values leading to optimum accuracy and efficiency on the Rutgers Sun HPC-10000 and the SDSC Cray T3E systems where the GADGET runs were produced. The softening length was chosen to be $h = 0.001$, smaller than both the BH gravitational radius $r_{gr} \equiv GM_{12}/\sigma_*^2 \approx 0.01$ and the separation corresponding to a hard binary, $r_h \equiv GM_{12}/8\sigma_*^2 \approx 0.0025$. We monitored the density profiles and Lagrangian radii of the stellar cusps as diagnostics sensitive to corruption of the bulk stellar distribution due to unacceptable levels of softening. Runs entering final selection exhibited no cumulative distortions on scales larger than $\sim h$. The total particle number used in the production run was $N = 2^{18} = 262,144$.

The late stages of the evolution were integrated using the direct-summation code NBODY6++ (Spurzem & Baumgardt, 1999). Conceived for the study of relaxation phenomena in globular clusters, NBODY6++ and its serial progenitor NBODY6 are the last and most complex codes in the NBODYx series of Aarseth (1999), employing the fourth-order Hermite scheme (Makino & Aarseth, 1992) as their primary integrator. The codes were written to facilitate the exact integration (no softening) of a large number of bodies with approximately equal masses. NBODY6++ gives particles adaptive block-individual time steps $\Delta t_n \propto 2^{-n}$ that are short for particles in dense regions and as much as 10^2 times longer for particles at the outskirts of the system. The Ahmad & Cohen (1973) scheme is used to select a subset of neighbors whose forces on the test particle are extrapolated at higher time resolution than those of the non-neighbors. Near encounters are treated using the two-body KS regularization scheme (Kustaanheimo & Stiefel, 1965) and its generalizations to systems with a few bodies, including the triple, quad and chain regularizations. Chain regularization is a systematic procedure for serializing the pairwise KS regularization in a group of not more than six bodies in close approach. NBODY6++ was developed to run on low-latency distributed memory architectures such as the Cray T3E. There is no domain decomposition: every processing node contains an identical copy of the whole dataset. Only the do-loops are broken into parallel sections; after every force calculation, an all-to-all broadcast scheme updates

TABLE 2
NBODY6++ INITIAL CONDITIONS

Label	Reduction in Particle Number	Truncation in Energy	N	M_{\bullet}/m_{\star}
GADGET	1×	1×	262,144	1,311
A2	2×	4×	32,768	655
A4	4×	4×	16,384	328
A8	8×	4×	8,192	164
B2	2×	4×	32,768	1,311
B4	4×	4×	16,384	655
B8	8×	4×	8,192	328

the particle sets. The treatment of binaries (regularization etc.) is not parallel. In the simulations conducted on the 272-node Cray T3E at the SDSC and the 64-node Sun HPC-10000 at Rutgers, NBODY6++ scaled well with the number of processors when the spread in time steps was moderate. When a few particles had much shorter time steps than others, the scaling was poor and serial integration on 666 MHz Alpha chips was found to be preferable.

One goal of our study was to identify any N -dependent features of the evolution, since real galaxies have much larger numbers of stars than accessible to direct simulation. To isolate the effects of varying N from other dependence (and also because NBODY6++ can not deal with particle numbers as large as 10^5 on currently available machines), we drew various random particle sets from the $N = 2^{18}$ GADGET integration and used these as initial conditions for the NBODY6++ runs. Half of the particles were iteratively removed and the masses of the remaining particles (except for the BHs) were doubled. We chose the time $t_0 = 10.6$ to select our reduced data sets; the separation of the BHs at this time is 0.072, substantially greater than their separation (~ 0.0025) at the time t_h when a hard binary forms.

In addition, to increase the effective resolution near the center, we sorted the data set by energy $E_i = m_{\star}[v_i^2/2 + U(\mathbf{r}_i)]$ and removed the upper 3/4 of all stars. While the new data sets were statistically distinct from the original, we expect to find unchanged dynamics in the cusps where the fractional perturbation from the removed stars is negligible. Combination of these two techniques led to a set of initial conditions for NBODY6++ labeled according to the formula “An” where n is the fractional reduction (Table 2).

In order to verify the accuracy of the tree code integrations, we continued the GADGET run for several dynamical times after t_0 and compared the results with the NBODY6++ integrations. Coincidence was found to persist until the separation r_{12} between the BHs was $\sim h$, at which point the BHs start to “see” each other’s spurious finite extent in the softened GADGET integrations (Figure 1). From that point on, the binary separation saturates with GADGET but continues to decrease with NBODY6++. We conclude that GADGET faithfully reproduces the dynamics of merging stellar cusps and BH binaries in regimes where the binary is soft.

NBODY6++ would seem to contain all of the machinery necessary for efficiently and accurately handling star-star, BH-BH and BH-star interactions. In fact, some of the greatest strengths of NBODY6++ in the context of glob-

ular cluster simulation were found to be weaknesses when the code is applied to systems with massive BHs. Chain regularization becomes impractical when some of the bodies are much more massive than others. In a model galaxy with $N = 10^6$ stars and a dense $\rho \sim r^{-2}$ stellar cusp harboring a $M_{\bullet}/m_{\star} = 10^3$ central BH, there are $\sim 10^3$ bodies whose orbits are largely determined by the BH’s potential. Although these stars satisfy the requirements for KS regularization (tiny mutual separations and short time steps), only the forces between the BH and the stars are significant; star-star encounters are energetically unimportant. However NBODY6++ is incapable of making this distinction. It will either try to generate a KS chain containing 10^3 bodies, which would defeat the purpose of regularization altogether and is beyond the present design; or, if one “turns off” the chain regularization, NBODY6++ will resort to a pairwise KS regularization whenever two stars come close, even if the motion of both stars is mostly in response to the force from a BH. In order to make the integrations go efficiently, it was sometimes necessary to remove “by hand” a few particles that were in tight orbits around one of the BHs.

Our simulations uniquely incorporate three features. 1. The galactic merger leading to the formation of the binary BH is carried out from a state in which both galaxies are reasonably isolated. 2. Both galaxies initially contain faithful realizations of steep stellar density cusps. 3. The BHs are present in the cusps at the outset in a kinematically consistent manner. Earlier studies have incorporated some of these features but never all of them. Makino & Ebisuzaki (1996) reported mergers of King models with $N = 16,384$ particles and central BHs with $M_{\bullet} \geq 1/64$. They conducted repeated mergers by recycling the merger products into initial conditions for successive mergers, after replacing the pair of BHs by a center-of-mass particle and reducing the number of particles by half. Makino & Ebisuzaki’s choice of King models as initial conditions made their galaxies poor representations of real stellar spheroids which always contain power-law density cusps; nor could they test the hypothesis that weak cusps are generated by the interaction of BHs and surrounding stars. Makino (1997) studied the evolution of a BH binary produced by a similar sequence of mergers, also using King-model initial conditions, but with a much larger maximum number of particles, $N = 262,144$. While the number of particles in Makino’s and our simulations is effectively the same, judging from the density profiles in Figure 4 of his paper, Makino’s initial conditions appear to be $\sim 10^2$

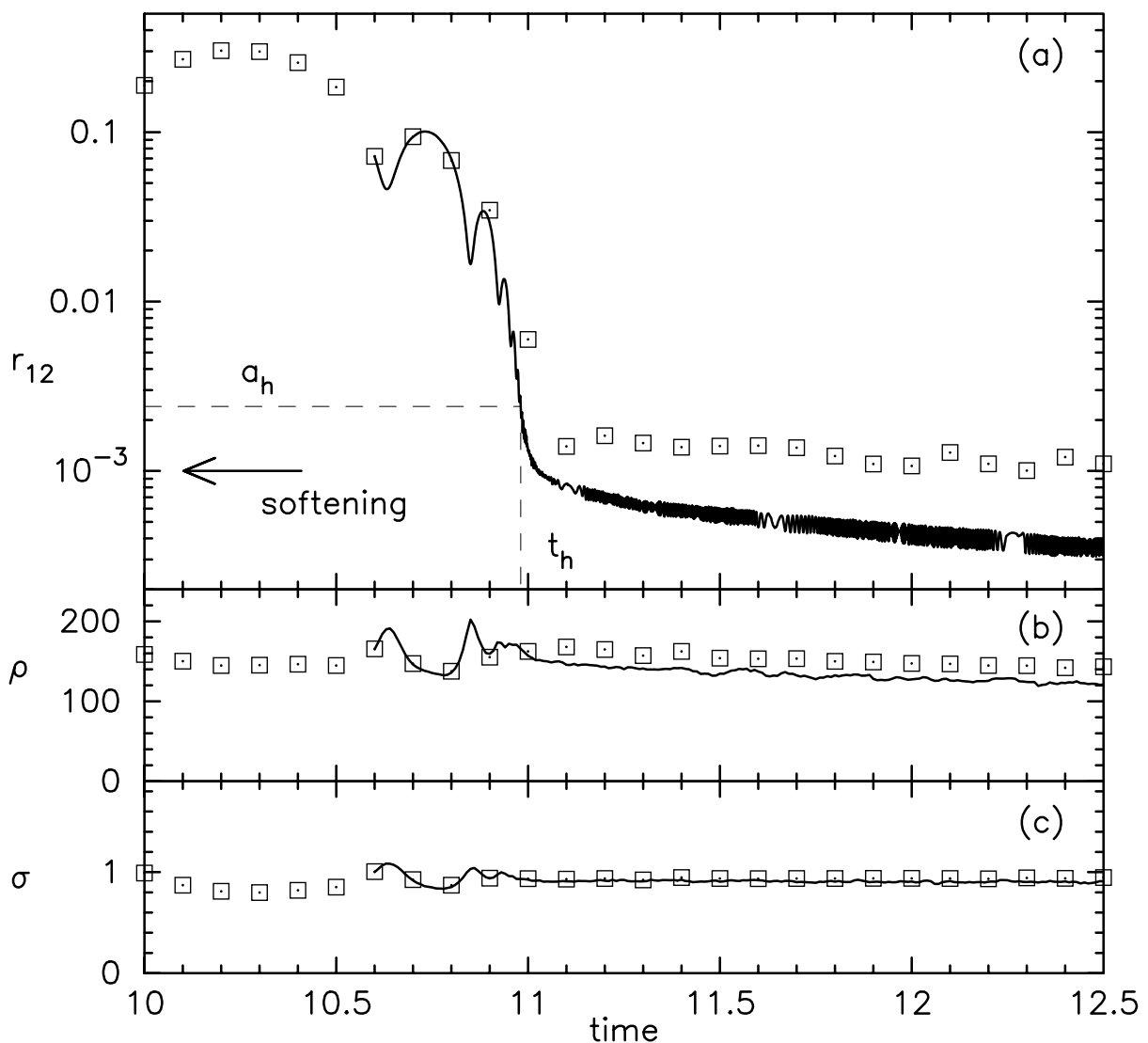


FIG. 1.— (a) Evolution of the separation between the BHs. Time is measured from the start of the simulation in units such that the crossing time in a single galaxy is ~ 2.2 . The BHs form a hard binary at $t \equiv t_h \approx 11.0$. Squares are from the 2.62×10^5 -particle integration with the tree code GADGET; the binary separation saturates at roughly the softening length h , marked by the arrow, in this simulation. Solid line is from the NBODY6++ integration A2 with $M_\bullet/m_* = 655$. NBODY6++ is able to follow the decay of the binary to arbitrarily small scales. (b) Stellar density as a function of time of stars separated by distance $r \leq 0.04$ from either of the BHs. (c) Stellar velocity dispersion within $0.01 \leq r \leq 0.04$ around each black hole.

times less dense than ours inside of the binary’s sphere of influence. Quinlan & Hernquist (1997) studied the evolution of a BH binary inside cuspy models with $\rho \sim r^{-1}$ and $\rho \sim r^{-2}$ and a wide range of BH masses and particle numbers, $N \leq 2 \times 10^5$. As discussed above, Quinlan’s code was unable to simulate an actual merger due to the limitations of its treatment of the mean field. All of the detailed results in their paper were derived from initial conditions consisting of a *single* galaxy into which two “naked” BHs were dropped from starting points located diametrically apart at the half-mass radius. This configuration is likely to produce substantial evolution of the cusp *before* the formation of the binary as the infalling BHs heat the stars; this can in fact be seen in their Figure 1, a plot of Lagrangian radii over time. As in the simulations of Makino & Ebisuzaki (1996) and Makino (1997), the initial density of stars around the BH binary in the simulation of Quinlan & Hernquist (1997) was much lower than in ours. This

difference will turn out to be consequential (§4).

Barnes (1999) presented simulations of mergers of identical galaxies with power-law cusps and no BHs. He used $N = 65,536$ bodies and a fixed time step, leap-frog scheme. Barnes showed that, outside of the softening length, power-law cusps as steep as $\gamma = 2$ are preserved by the merger. We find an analogous result (§3).

Our simulations, being purely dynamical, preclude any non-dynamical processes such as gas-driven dissipation that might act to accelerate the binary’s decay. Decay might also be enhanced by dynamical processes that we do not include, e.g. the passage of a star cluster, gas cloud or third supermassive BH through the nucleus. How different is the signature on the stellar distribution of a binary that coalesces immediately after its formation? We address this question by shadowing each simulation introduced above with another where the BH binary is replaced by a single BH of mass $2M_\bullet$ at time $t = t_0 + 0.4 = t_h$. This yielded

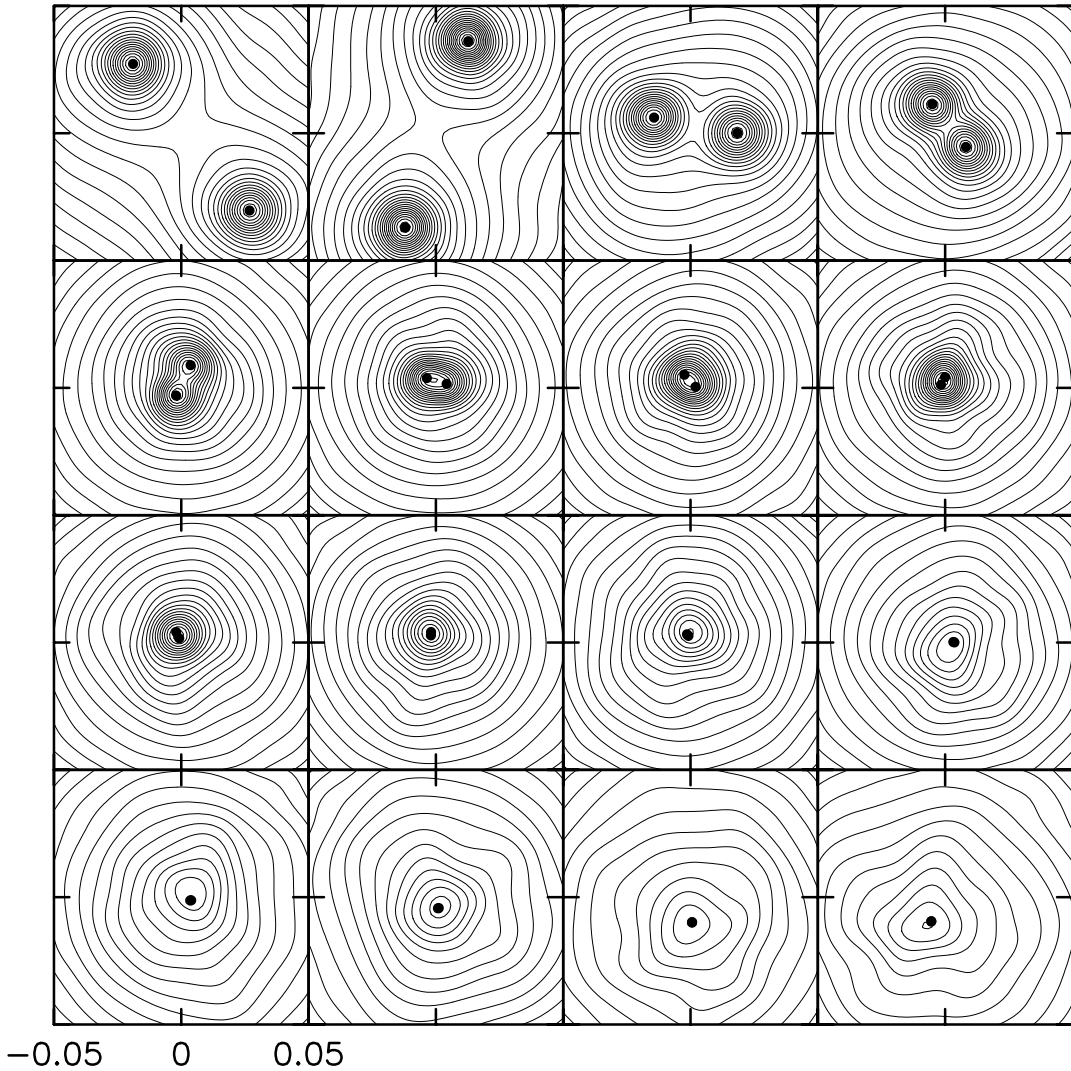


FIG. 2.— Projected density contours for the run A2 (NBODY6++) with $M_{\bullet}/m_{\star} = 655$. The orbital motion of the BHs (positions indicated by filled circles) is clockwise in the plane of the figure. First row: $t = 10.67, 10.8, 10.98, 10.91$. The two cusps spiral-in under the influence of dynamical friction. Second row: $t = 10.94, 10.95, 10.96, 10.97$. The two cusps merge into one. The final density profile is similar to that of the initial stellar systems. Third row: $t = 10.98, 11.0, 11.1, 11.6$. The density of the newly-formed cusp drops rapidly as the BH binary transfers energy to the stars. Fourth row: $t = 12.6, 13.6, 16.6, 18.6$. Density continues to drop as the BH binary ejects stars.

a second set of initial conditions labelled as “Bn” in Table 2.

3. CUSP COALESCENCE

We divide the evolution into two regimes, before and after the formation of a hard BH binary, and discuss the first regime in this section. The two regimes correspond approximately, but not exactly, to the intervals before and after the start of the NBODY6++ integrations; as discussed above, these integrations were begun when the BHs were still a few softening lengths apart and had not yet formed a tightly bound pair.

As Quinlan (1996) notes, there are many ways to define a “hard” binary. The standard definition is energy based: a binary is hard if its binding energy exceeds the typical particle kinetic energy, $|E_b| \gg 3m_{\star}\sigma^2/2$. This definition is inapplicable to the case of massive BH binaries since the binary-to-stellar mass ratio scales with N while σ is independent of N ; according to this definition, a very massive BH binary would always be hard if bound and soft oth-

erwise. The famous law of Heggie (1975), asserting that hard binaries evolve toward even harder states, suggests a second definition of hardness. While a viable distinguishing criterion for hard binaries in star clusters, Heggie’s law fails to capture the transition between two different processes—dynamical friction and mass ejection—that *both* tend to drive a massive binary to an ever-harder state in our simulations.

We therefore followed the suggestion of Hills (1983) and Quinlan (1996) and defined a “hard” binary in terms of its orbital velocity. The orbital velocity of each BH in a circular-orbit binary is $v_c^2 = GM_{12}/4a$ with M_{12} the combined mass of the two BHs and a their separation. We defined the critical separation at which a binary becomes “hard” as

$$a_h = \frac{GM_{12}}{8\sigma_{\star}^2} \quad (4)$$

corresponding to $v_c = \sqrt{2}\sigma_{\star}$. In model units, $a_h \approx 2.5 \times 10^{-3}$ and the binary separation first falls below a_h at $t \equiv t_h \approx 11.0$ (Figure 1). “Subsonic” massive binaries,

$a \gg a_h$, harden by dynamical friction acting on each BH (and its associated cluster) individually; a “supersonic,” or hard, binary behaves like a structureless point mass under the action of dynamical friction but can capture stars and eject them at much higher velocity, thereby increasing its hardness. Quinlan (1996) notes that this definition of hardness is roughly equivalent to the statement that a hard binary hardens at a constant rate. We found this to be true (§4); the definition (4) is also a natural one in the sense that the character of the binary’s evolution, and the evolution of its surrounding stellar cluster, were found to undergo qualitative changes when a dropped below $\sim a_h$, corresponding to the onset of mass ejection by the binary.

The merging of the stellar cusps for run A2 is illustrated in Figure 2. This figure makes manifest that the BHs remain closely associated with their initial stellar cusps during every stage of the merger, up to and including the point when the two cusps merge into one at $t \approx t_h$. A consequence is that *dynamical friction brings the BHs together much more rapidly than if they were “naked,” since their effective mass is the mass of the cluster of stars bound to them.*

We can check this assertion by comparing the orbital decay rates for an isolated BH and that embedded in a stellar cusp. The decay rate for an isolated BH on an approximately circular orbit is given by (Binney & Tremaine, 1987)

$$\begin{aligned} \frac{da}{dt} &= -\frac{\text{erf}(1) - \text{erf}'(1)}{\sqrt{2}} \frac{GM_\bullet}{\sigma_* a} \ln \Lambda \\ &\approx -0.302 \frac{GM_\bullet}{\sigma_* a} \ln \Lambda \end{aligned} \quad (5)$$

where a denotes separation between two BHs. With $M_\bullet = 0.01$, $a = 0.1$ and $\sigma_* \approx 2^{-1/2}$, the decay rate is estimated at $da/dt \approx -0.043 \ln \Lambda$. Note that for $a > 0.1$ the formula yields even lower rate of decay. As for the Coulomb logarithm, it can be written in terms of the ratio of the maximum and the minimum impact parameter $\ln \Lambda = \ln(p_{\max}/p_{\min})$ where it is a standard choice to select the gravitational radius of the isolated “test particle” for the latter, $p_{\min} = GM_\bullet/\sigma_*^2$. In lack of a canonical choice for p_{\max} , we equate it to the orbital radius, which implies $\ln \Lambda = \ln 5.0 \approx 1.6$ and thus $da/dt \approx -0.07$ (but see also Appendix A). The predicted decay rate is a factor of ~ 6 smaller than the rate of in-spiral $da/dt \approx -0.43$ we measured in the GADGET run in the interval $9.0 \leq t \leq 11.0$ preceding the formation of hard binary, in this interval $a(t) = 0.78 - 0.43 \times (t - 9.0)$ is a good fit.

We compare the isolated particle estimate with an estimate of how rapidly dynamical friction would act to bring together two overlapping spheres with $\rho \sim r^{-2}$ density profiles. Let the spheres each have mass \mathcal{M} and density $\rho = \sigma_*^2/2\pi Gr^2$ inside a radius a . When the separation a between their centers is much larger than the BH radius of influence, or equivalently $\mathcal{M}(a) \gg M_\bullet$, we can ignore the BHs. Then $\mathcal{M}(a) = \sigma_*^2 a/G$ and the circular velocity is $v_c^2(a) = \sigma_*^2/2$. The dynamical friction force acting on one of the spheres is given by

$$\begin{aligned} \langle \Delta v_{\parallel} \rangle &= -\frac{4\pi G^2 \mathcal{M} \rho \ln \Lambda F(v)}{v^2}, \\ F(v) &= \text{erf}(x) - x \text{erf}'(x) \end{aligned} \quad (6)$$

(Chandrasekhar, 1943), where $\ln \Lambda$ is the Coulomb logarithm and $x = v/\sqrt{2}\sigma_*$. Setting $v = v_c = \sigma_*/\sqrt{2}$ gives $F = 0.0811$. Taking for ρ the density of either sphere at a distance a from its center, we find

$$\langle \Delta v_{\parallel} \rangle \approx -\frac{0.324 \sigma_*^2 \ln \Lambda}{a}. \quad (7)$$

Equating the torque produced by this acceleration with the change of orbital angular momentum J , and writing

$$\frac{dJ}{dt} = \frac{dJ}{da} \frac{da}{dt} = \frac{\sigma_*^3 a}{\sqrt{2}G} \frac{da}{dt}, \quad (8)$$

gives

$$\frac{da}{dt} \approx -0.23 \sigma_* \ln \Lambda. \quad (9)$$

Just outside of the BHs’ sphere of influence, $\sigma_* \approx 1$ (Figure 1b). For $\ln \Lambda$ we again take ~ 1.0 , giving $da/dt \approx 0.24$. This result still falls short of the observed value ($da/dt \approx -0.43$) by a factor of ~ 2 .

A detailed integration (Appendix A) taking into account the shape and the finite extent of both spheres yields the rate $\langle \Delta v_{\parallel} \rangle \approx -1.50 \sigma_*^2/a$ (note the absence of $\ln \Lambda$), therefore $da/dt \approx -1.06$. This result, however, is sensitive to the choice of tidal radius outside of which the spheres are indistinguishable; estimation of this radius is difficult in part due the large orbital eccentricity of the galaxies in the simulation.

Remarkably, the density structure of the merged galaxy *just after* formation of a hard binary is essentially identical to that of the initial stellar systems at radii $r \gtrsim a$. This is illustrated in Figure 3. Homology following a merger was

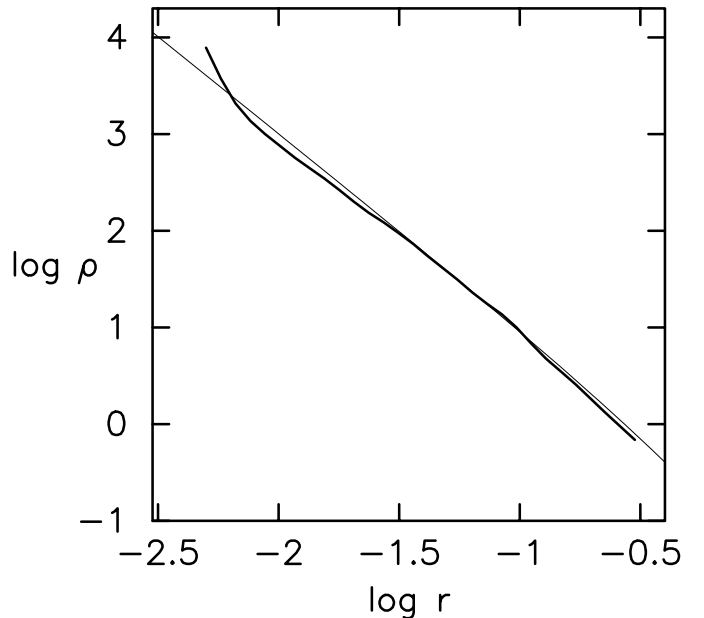


FIG. 3.— Radial density profiles of the pre-merger galaxies (thin curve) and at time $t \approx t_h = 10.96$ (thick curve) when the binary separation equals $r_{12} = 6.5 \times 10^{-3}$. The pre-merger density was multiplied by the factor $(Mr_0^{-3})_{\text{new}}/(Mr_0^{-3})_{\text{old}} \approx 0.53$ to bring it to the scale of the post-merger galaxy. The two galaxies have merged into a single galaxy that is nearly homologous with the initial galaxies on scales $r \gtrsim r_{12}/2$. Shortly after this time, the central density drops as the binary heats the core.

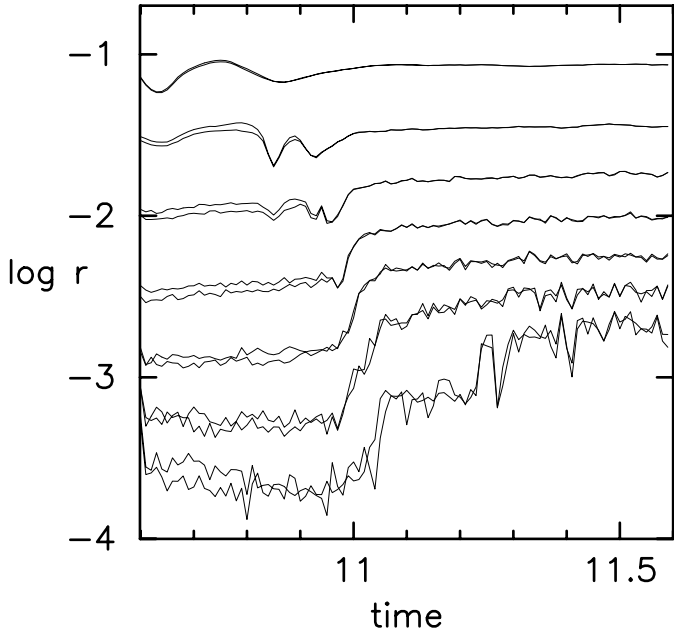


FIG. 4.— Lagrangian radii around each BH in the first time unit of the NBODY6++ run A2. From bottom to top, the radii enclose 10^{-4} , $10^{-3.5}$, 10^{-3} , $10^{-2.5}$, 10^{-2} , $10^{-1.5}$ and 10^{-1} in units of the mass of one galaxy before the merger.

found also by Barnes (1999) in spherical galaxies without BHs. In our simulations, however, the homology is short-lived. The formation of a hard BH binary at $t \approx t_h$ is followed by a sudden drop in the stellar density within the binary’s gravitational sphere of influence, $r \lesssim 0.01$. This is clearly seen in Figure 4, a plot of Lagrangian radii, and also in the density contour plot of Figure 2. (The drop in density is not so apparent in Figure 1b because the density plotted there is an average over a radius of 0.04, and little net change in density occurs within this radius – see Figure 4.) In effect, the steep cusp that was present immediately after formation of the BH binary is destroyed in little more than the local crossing time.

What is responsible for the rapid destruction of the cusp? Two possible, and closely related, mechanisms are deposition of energy into the stars by dynamical friction acting on the BHs individually; and ejection of stars that exchange energy with the BH binary. Neither process is well defined in this regime where the BH binary is neither very hard nor very soft. Nevertheless we can write approximate expressions for the rate at which energy is transferred to the stars by the two mechanisms, by assuming either that the BHs are moving independently of each other, or as members of a tight binary; of course neither assumption is strictly satisfied.

In the first case, dynamical friction would extract energy from the two BHs at a rate

$$\langle \Delta E \rangle = 2Mv \langle \Delta v_{||} \rangle = -\frac{8\pi G^2 M^2 \rho \ln \Lambda F(v)}{v} \quad (10)$$

(cf. equation 6). Setting $v = \sqrt{2}\sigma_*$, our definition for the onset of a hard binary, gives $F = 0.43 \approx 1/2$ and

$$\langle \Delta E \rangle \approx -2\sqrt{2}\pi G^2 M^2 \rho \sigma_*^{-1} \ln \Lambda. \quad (11)$$

The alternate mechanism, hardening of the binary by mass

ejection, produces energy at the rate

$$\frac{dE}{dt} = \frac{G^2 M^2 \rho H}{2\sigma_*} \quad (12)$$

(Hills, 1983; Mikkola & Valtonen, 1992; Quinlan, 1996), where H is the dimensionless hardening rate; $H \approx 15$ in the limit of a very hard, equal-mass binary and drops to ~ 10 for a binary with $a = a_h$. Thus, *both* mechanisms predict an energy deposition rate that can be written as

$$\left| \frac{dE}{dt} \right| = CG^2 M^2 \rho \sigma_*^{-1} \quad (13)$$

with $C \approx 5$; we have taken $\ln \Lambda \approx 0.5$ (Appendix A).

The energy of the binary when $a = a_h$ is $E = -2M\sigma_*^2$, so the characteristic time over which either process extracts energy is $\sim 2C^{-1}\sigma_*^3/G^2M\rho$. Computing ρ using the mass within $r = 0.01$ at $t = t_h$ gives an energy extraction time scale of ~ 0.2 in model units. This is quite comparable to the time associated with the jump in Lagrangian radii of Figure 4. We note also that the energy extracted from the binary in this time, $E \approx 2M\sigma_*^2 \approx 0.02$, is comparable to the energy in stars within a radius of ~ 0.01 . This too is consistent with the changes in Lagrangian radii shown in Figure 4.

We conclude that the sudden disruption of the steep cusp is attributable to transfer of energy from the BHs into the surrounding stars as the BHs form a hard binary. We emphasize again the *rapidity* of this process, which earlier analyses have overlooked. If a galaxy’s cusp is to avoid this fate, some mechanism must extract energy from the

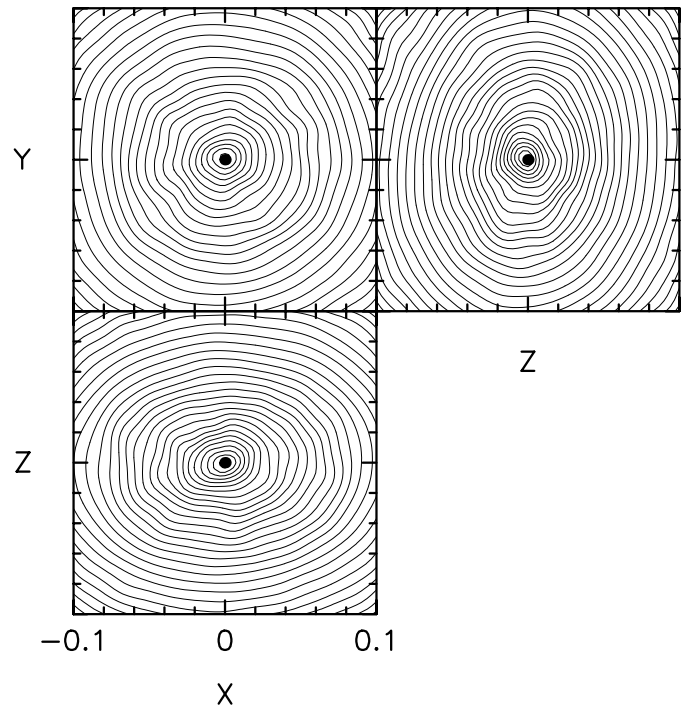


FIG. 5.— Isophotes in the run A2 in three projections. Black dots show the location of the BH binary; separation between the BHs at this time is $a \sim 1.5 \times 10^{-4}$. 100 snapshots of the nucleus were superposed in the interval $18.1 \leq t < 19.1$. The merger remnant is approximately axially symmetric with an edge-on ellipticity of $\epsilon \approx 0.25$.

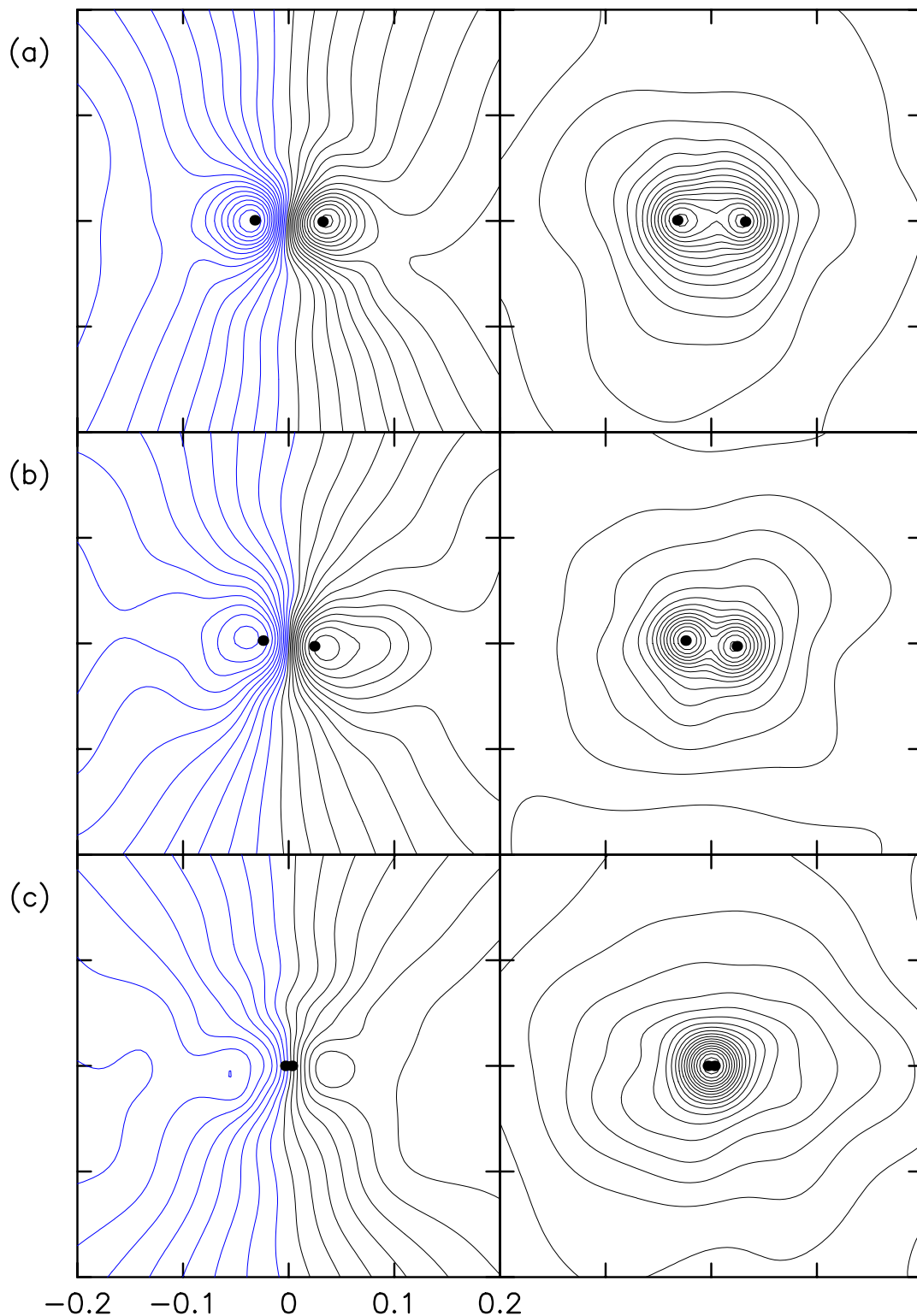


FIG. 6.— 2D kinematics of the merging cusps for the run A2. View is in the plane of the merger from a direction perpendicular to the line connecting the two BHs. Left panels show the mean line-of-sight velocity; blue contours indicate approaching stars. Right panels are the line-of-sight velocity dispersion. In all panels the contours are separated by 0.038. (a) $t = 10.66$; (b) $t = 10.82$; (c) $t = 10.96$. The BHs remain centered on the velocity dispersion peaks but move inward with respect to the peak of the rotational velocity.

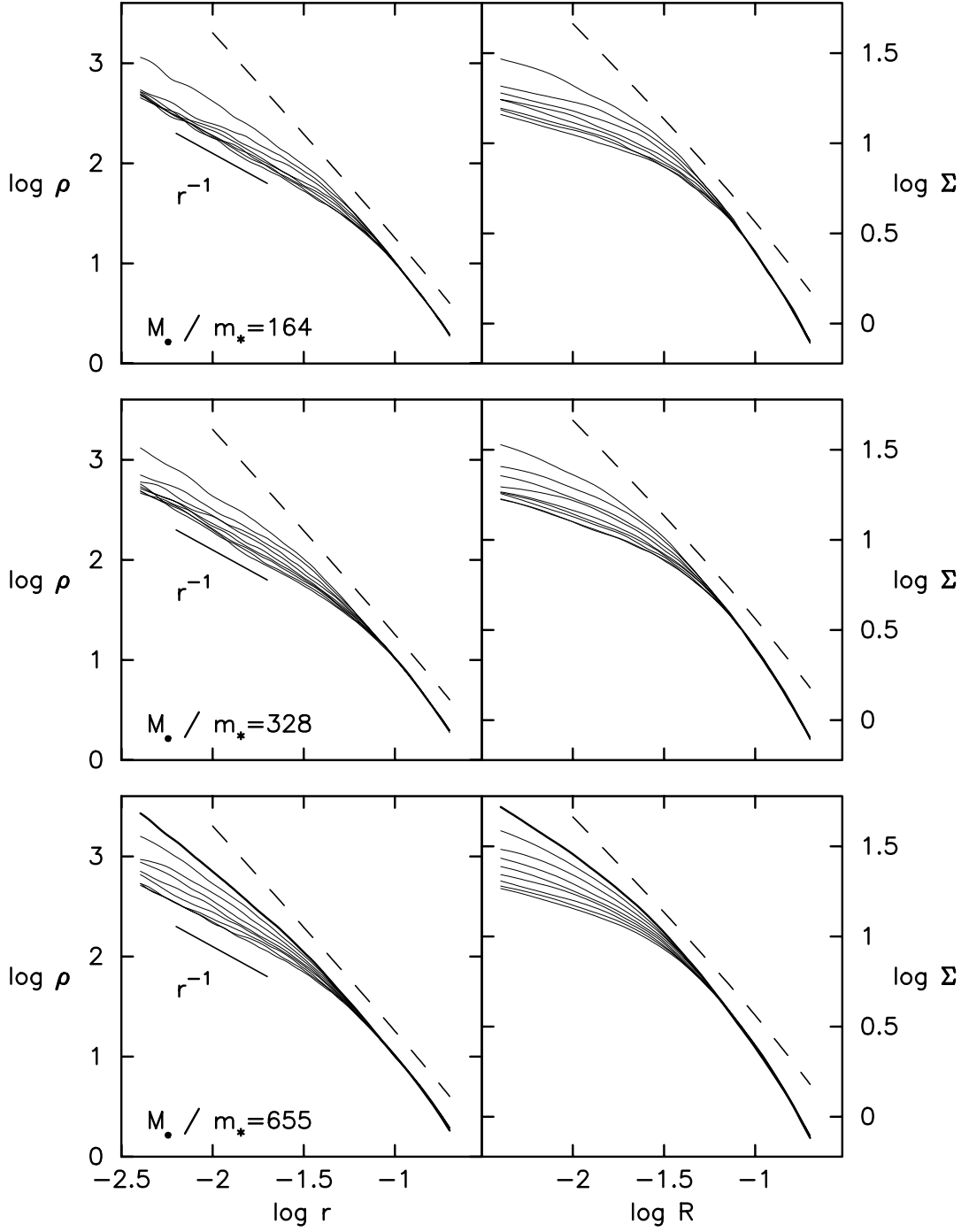


FIG. 7.— Spatial density profiles (left column) and projected density profiles (right column). In each panel, starting at time $t = 11.0$, the profiles are recorded from top to bottom at intervals $\Delta t = 1.0$, each of them an average obtained by superposing 100 snapshots. Top row, run A8; middle row, run A4; bottom row, run A2. Thick lines represent the run B2 with one BH; the merger remnant would have this profile if the BHs coalesced at $t = t_h$. Dashed lines are profiles of the original galaxies multiplied by an arbitrary factor. The merger remnant has a $\rho \sim r^{-1}$ density cusp which projects to a core profile with continuous curvature.

binary BH on a time comparable to the local dynamical time, before it is able to exchange energy with the stars.

The density near the BHs continues to drop at later times although more slowly, as the BH binary gradually decays. We discuss this process in more detail below.

The large-scale kinematical evolution of the merger is illustrated in Figure 6. The general character of these plots is similar to what is seen in simulated mergers of equal-mass galaxies without BHs (e.g. Bendo & Barnes (2000)), with the highest mean rotational velocities in the plane of the merger and a roughly cylindrical rotation pattern elsewhere. The peaks in the rotational velocity initially correspond with the locations of the BHs, but dynamical friction causes the BHs to move inward from these peaks at a time $t \approx 10.75$. Velocity dispersions, on the other hand, remain peaked on the BHs at all times consistent with the fact that the BHs remain centered on their cusps (Figure 2). At the time of formation of the hard BH binary, $t = t_h \approx 11.0$, the merger remnant is mildly rotating with a peak line-of-sight rotational velocity of $v/\sigma_* \approx 0.58$ at a distance ~ 0.04 from the center (using $\sigma_* \approx 0.6$ at distance 0.2), although bulk fluctuations at 20% level in the rotation field persist on scales $r \lesssim 0.1$. The ellipticity of isophotes shown in Figure 5 is $\epsilon \approx 0.25$ which falls on the isotropic oblate rotator relation $v/\sigma_* = \sqrt{\epsilon/(1-\epsilon)}$.

Evolution of the stellar density profiles is shown in Figure 7. Profiles were computed from the N -body positions using a nonparametric kernel routine based on the algorithms in Merritt & Tremblay (1994); details are given in Appendix B. Each profile is an average over several snapshots; they are separated by $\Delta t = 1.0$ starting at $t = t_h = 11.0$. The evolution of $\rho(r)$ becomes more regular as N is increased, due probably to the smaller random motion of the binary for larger N . Considerable evolution occurs in the interval $t_h - 1 \lesssim t \lesssim t_h + 1$ when the BHs form a hard binary, as discussed above. A break appears in the profiles at $t \approx t_h$ where the outer, $\rho \sim r^{-2}$ profile turns over to a shallower inner dependence; the inner profile is well approximated as a power-law as well, with slope $d \log \rho / d \log r \approx -1$ that gradually decreases with time. In projection, this weak power-law cusp produces a core-like profile with continuously varying slope. Hence this galaxy would be classified as a “core galaxy” for $t \gtrsim t_h$ (Lauer et al., 1995); we note that core galaxies also show weak power-law cusps on deprojection (Merritt & Fridman, 1996). We defined the “break radius” R_b as the radius where the second derivative of $\Sigma(R)$ on a log-log plot reaches a minimum; this definition is consistent with the more common one based on fitting of a parametric form to the surface brightness profile (e.g. Lauer et al. (1995)). We defined r_b in the same way, as the break radius corresponding to the space density profile $\rho(r)$. Values of R_b and r_b at several different times are given in Table 3.

Our simulations are the first to demonstrate that weak, power-law cusps – corresponding to what are commonly called “core” or “cuspy-core” galaxies – can be generated by the merger of galaxies with steep cusps, or “power-law” galaxies. Since core galaxies are systematically brighter than power-law galaxies, it is natural to suppose that weak cusps have their origin in mergers. We will explore this hypothesis in more detail below, after discussing the further evolution of the density profiles that takes place as the

BH binary slowly decays. Here we discuss one problem with the hypothesis, and a possible resolution. Even some moderately bright elliptical galaxies ($M_V \approx -22$) exhibit steep cusps, even though these galaxies have certainly experienced mergers in the past. How did these galaxies avoid the rapid cusp destruction that takes place in our simulations?

A possible answer is suggested by Figures 3 and 7. Immediately after the merger, at $t \approx t_h$, the density profile is briefly almost homologous with the initial profile, with a steep, $\rho \sim r^{-2}$ cusp. If some mechanism could induce a rapid coalescence of the BHs at this time, before the BH binary was able to exchange energy with the stars, the steep cusp might avoid disruption. We tested this idea using our runs in which the two BHs were artificially combined into one at $t = t_0 + 0.4 = 11.0$. The test was successful; the density profile after coalescence of the binary (shown as the heavy line in the bottom panels of Figure 7) is indeed very close to the initial profile and remains so indefinitely. We discuss below (§6) whether any mechanism might exist for inducing such a rapid coalescence, and why it should preferentially be active in low-luminosity galaxies.

4. EVOLUTION OF THE BLACK-HOLE BINARY AND ITS EFFECT ON THE STRUCTURE OF THE NUCLEUS

4.1. Physical Processes

The evolution of a massive BH binary in a galactic nucleus has been discussed by a number of authors. We begin by summarizing that work here and listing the physical processes that govern the evolution of the binary and its effect on the surrounding stars.

1. *Hardening of the binary.* Stars that pass within a distance $\sim a$ of the binary, with a the binary’s semi-major axis, experience a gravitational slingshot and are ejected with velocities $v_{ej} \approx V_{bin} \equiv \sqrt{GM_{12}/a}$ (e.g. Hills & Fullerton (1980)); V_{bin} is the relative velocity of the two BHs if their orbit is circular and $M_{12} = M_1 + M_2$ is the total mass of the binary. In a fixed stellar background, this leads to hardening at a rate

$$\frac{d}{dt} \left(\frac{1}{a} \right) = \frac{G\rho}{\sigma_*} H \quad (14)$$

and the rate of energy extraction from the binary is

$$\frac{dE_b}{dt} = \frac{G^2 M_{12}^2 \rho}{8\sigma_*} H \quad (15)$$

with H a dimensionless hardening rate. Here ρ and σ_* are the mass density and 1D velocity dispersion of the stars. Equations (14) and (15) are derived from a model in which the stellar density is assumed uniform and the gravitational field from the stars is ignored; gravitational focusing by the BH binary is incorporated but not the influence of the stellar potential on stellar orbits. The dimensionless hardening rate H is a function of the hardness of the binary, measured for instance by V_{bin}/σ_* , as well as the binary’s mass ratio M_1/M_2 and eccentricity. For an equal-mass, circular-orbit binary, H varies from ~ 15 for an infinitely hard binary to ~ 2.0 for $V_{bin}/\sigma_* = 1$ (Quinlan, 1996).

Stellar encounters also modify the binary’s orbital ec-

centricity e . The eccentricity growth rate,

$$K = \frac{de}{d \ln(1/a)}, \quad (16)$$

is negligible for $V_{bin}/\sigma \approx 1$ and increases to a maximum of ~ 0.2 for an equal-mass binary with $e \approx 0.7$ and $V_{bin}/\sigma \gtrsim 20$ (Mikkola & Valtonen, 1992; Quinlan, 1996). Changes in eccentricity are potentially important because the rate of orbital energy loss due to gravitational radiation grows steeply for $e \rightarrow 1$ (cf. equation 22), hence an eccentric binary will coalesce sooner than a circular one with the same a .

2. *Mass ejection.* The binary ejects mass at a rate

$$J = \frac{1}{M_{12}} \frac{dM_{ej}}{d \ln(1/a)} \quad (17)$$

where $J \approx 1$ is nearly independent of $(M_1/M_2, a)$ for $a \ll a_h$ and drops with decreasing hardness of the binary (Mikkola & Valtonen, 1992; Quinlan, 1996). Ignoring the variation of J with V_{bin} , one can integrate equation (17) to obtain

$$M_{ej} \approx J M_{12} \ln(a_{ej}/a) \quad (18)$$

where it has been assumed that mass ejection begins when $a = a_{ej}$; we expect $a_{ej} \approx a_h$. Thus the binary ejects of order its own mass in shrinking from $a = a_{ej}$ to $a = a_{ej}/2$. If the binary's mass is not negligible compared with the mass of the pre-existing nucleus, the stellar density near the binary will drop as the decay proceeds, causing the hardening rate to also drop (equation 14).

3. *Brownian motion.* The binary exhibits Brownian motion due to momentum imparted by encounters with stars. A single particle of mass M in statistical equilibrium with a Maxwellian field of light scatterers with masses $m_* \ll M$ will exhibit an average speed determined by equipartition of energy,

$$M_{12} \langle v^2 \rangle = 3 m_* \sigma_*^2, \quad (19)$$

and its radius of wandering r_w will be given by

$$\langle r_w^2 \rangle \approx \frac{\langle v^2 \rangle}{G \rho} \quad (20)$$

where ρ is a mean density averaged over the wandering region. These relations ignore any reaction of the background particles to the motion of the massive object. Corrections also apply if the massive object is a binary, which receives larger kicks than a point mass from ejected stars. The speedup is at most a factor of ~ 2 for a very hard binary in a nucleus with a steep density profile (Merritt, 2001). Brownian motion is potentially important because it allows the binary to interact with a larger pool of stars than if it were fixed at the center of the potential, thus prolonging its decay. However the amplitude of the wandering in an N -body simulation is likely to be much larger than in a real galaxy due to the unphysically small value of M_{12}/m_* in the simulations. Brownian motion may also help to scatter stars into the binary's sphere of influence by introducing a complex time dependence into the gravitational potential felt by the stars.

4. *Loss-cone refilling.* Eventually the binary will eject most or all of the stars which can come within a distance

$\sim a$ of it. If the binary wanders over a distance $r_w > a$, this will happen when it has ejected all stars whose pericenters lie within a distance $\sim r_w$ from the galaxy center. Once the density of stars in the vicinity of the binary drops to zero, the binary's decay will stall, unless some process can refill the "loss cone." One possibility is infall of a third BH, gas cloud, dwarf galaxy or other massive object that can perturb the stellar orbits. In the absence of such dramatic events, ordinary two-body relaxation will scatter stars into the binary's sphere of influence. The associated feeding rate is

$$\frac{dM_{scat}}{dt} \approx \frac{M(r_{max})}{t_r(r_{max}) \ln(r_{max}/r_w)} \quad (21)$$

where $M(r)$ is the stellar mass within r and t_r is the star-star relaxation time; r_{max} is the radius at which the rate of scattering into the loss cone peaks, typically of order r_{gr} (Shapiro, 1985).

5. *Gravitational radiation.* If the decay of the binary continues sufficiently far, emission of gravitational radiation will eventually become the dominant source of energy loss. The gravitational radiation time scale t_{gr} is

$$t_{gr} = \left| \frac{\dot{a}}{a} \right|^{-1} = \frac{5}{64} \frac{c^5 a^4}{G^3 M_{12}^3} F(e) \quad (22)$$

where the factor $F(e)$ contains the eccentricity dependence:

$$F(e) = \frac{(1 - e^2)^{7/2}}{1 + \frac{73}{24}e^2 + \frac{37}{96}e^4} \quad (23)$$

(Peters, 1964). The dependence of t_{gr} on e is weak for small e ; $F(0) = 1$ and $F(0.5) \approx 0.205$. The decay rate from gravity wave emission matches that from stellar ejection when

$$a^5 = a_{crit}^5 \equiv \frac{64}{5FH} \frac{G^2 M_{12}^3 \sigma}{c^5 \rho}. \quad (24)$$

The right hand side of this expression is difficult to evaluate *ab initio* since ρ will be strongly affected by stellar ejection during the binary's decay. However semi-analytic models for the combined evolution of the binary and the nucleus (Merritt, 2000) suggest that $a_{crit} \approx 10^{-2} a_h$. Equation (18) then implies that the binary must eject roughly four times its mass in stars in order to achieve gravitational radiation coalescence. We do not include an energy sink term corresponding to gravitational radiation in our simulations but use equation (24) to estimate when coalescence would occur.

A crucial question when interpreting N -body simulations is the dependence of the results on N . Real galaxies have nuclei with $N \approx 10^7$, much greater than the particle numbers amenable to computer simulation. Fortunately, the two processes that most directly affect the evolution of a BH binary in a galactic nucleus – hardening and mass ejection – have rates that depend only on the local *density* of stars, not on their masses. The rates at which these two processes occur in our simulations should therefore reflect their rates in galaxies whose mass distributions are similar to those in our models.

However both the Brownian motion of the binary and the refilling of the binary's loss cone by two-body encounters are N -dependent processes, and their importance in

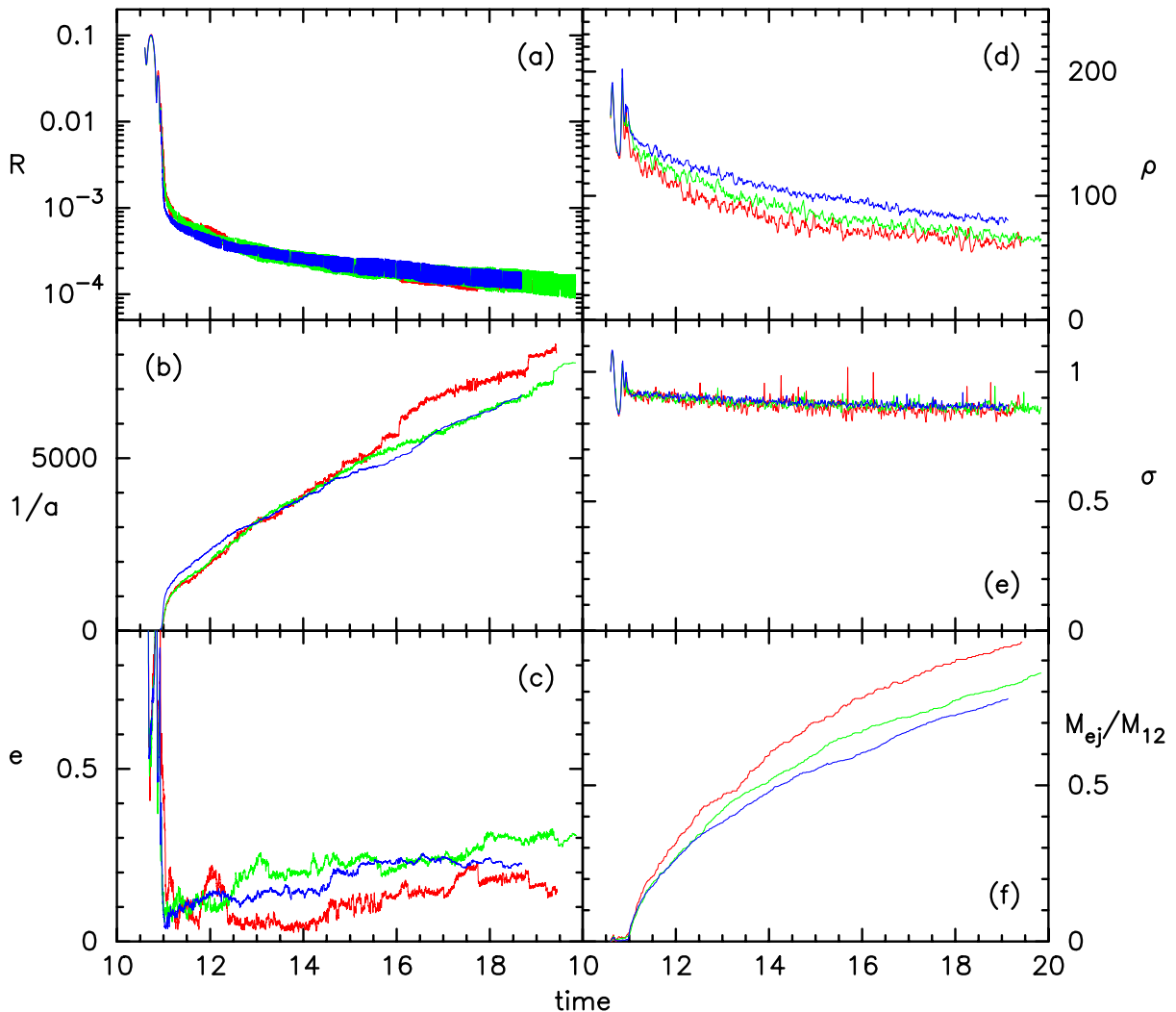


FIG. 8.— Evolution of the BH binary and response of the stellar environment. Red curves: run A8 ($M_{\bullet}/m_{*} = 164$); green curves: run A4 ($M_{\bullet}/m_{*} = 328$); blue curves: run A2 ($M_{\bullet}/m_{*} = 655$). (a) Separation between the BHs; (b) inverse semimajor axis $a^{-1} \equiv -2E_b/GM_{\bullet}^2$; (c) orbital eccentricity. When calculating a and e before t_h , we corrected the BH masses by adding the mass in stars bound to each of them. (d) Average density of stars within $r \leq 0.04$ from either of the BHs; (e) velocity dispersion in a region $0.01 \leq r \leq 0.04$ from either of the BHs; (f) total mass of stars ejected by the binary, in units of the binary mass.

our simulations is expected to be much greater than in real galaxies. The binary’s wandering radius scales as

$$r_w \propto \sqrt{\frac{m_{*}}{M_{12}}} \propto N^{-1/2} \quad (25)$$

while the rate of scattering of stars into the loss cone varies as

$$\frac{dM_{scat}}{dt} \propto t_r^{-1} \propto m_{*} \propto N^{-1} \quad (26)$$

where the N -dependence of the Coulomb logarithm has been ignored. These “second order” effects do not directly influence the binary’s evolution but they do determine how large a supply of stars is available to the binary and hence how long its orbit can continue to decay. For instance, a wandering binary can interact with a larger pool of stars than a binary that is stationary.

In a real galactic nucleus where N is very large, Brownian motion and two-body relaxation would be expected to be almost negligible. (Exceptions might occur in nuclei where the gravitational potential is very lumpy, due

to giant molecular clouds, star clusters, additional massive BHs etc.) An obvious inference, drawn by several authors (Valtonen, 1996; Merritt, 2000; Zier, 2000; Gould & Rix, 2000), is that a massive BH binary should rapidly eject those stars whose orbits bring them within its sphere of influence, after which the binary separation should cease to change. These arguments are not air-tight however because of the complicated way in which the various physical processes interact. For instance, mass ejection lowers the density of stars, causing the hardening rate to drop (equation 14), but the reduction in density leads to an increase in the wandering radius (equation 20) allowing the binary to move out of the low-density region into a region of higher density where it can continue interacting with stars.

4.2. Hardening rate and mass ejection

Figure 8 summarizes the evolution of the BH binary in our simulations. At any given time, the binary can be described by its semimajor axis a , its eccentricity e , the direction of its orbital angular momentum vector $\hat{\mathbf{n}}$, and

TABLE 3
MEASURED N -BODY PARAMETERS

Quantity	Time	A8	A4	A2
r_b	11	0.038	0.036	0.035
r_b	15	0.049	0.048	0.035
r_b	19	0.087	0.086	0.056
R_b	11	0.036	0.033	0.026
R_b	15	0.053	0.068	0.030
R_b	19	0.080	0.072	0.063
da^{-1}/dt	[11, 14]	970	920	750
da^{-1}/dt	[16, 19]	660	540	690
H	13	9.3	7.9	5.8
H	17	8.1	6.3	6.8
K	> 11.6	0.70	0.13	0.13
a_{ej}	> 12	0.00098	0.00085	0.00074
a_{ej}	> 13	0.00085	0.00074	0.00070
a_{ej}	> 14	0.00090	0.00071	0.00070
J_∞	> 12	0.45	0.45	0.46
J_∞	> 13	0.49	0.49	0.48
J_∞	> 14	0.48	0.51	0.48
σ_{12}	13	0.087	0.064	0.034
σ_{12}	16	0.085	0.061	0.032
σ_{12}	19	0.081	0.058	0.030
r_w	[11, 19]	0.028	0.011	0.0084

the position and velocity of its center of mass. The binary’s hardening rate da^{-1}/dt (Figure 8b) is nearly constant with time following the “knee” at $t \approx t_h$ when the binary first becomes hard. Minute fluctuations in $1/a$ reflect perturbations in the binary’s binding energy due to stars tightly bound to one of the BHs; sudden jumps indicate times when the binary ejects a single star. At low N , the discreteness of individual ejection events induces statistical fluctuations in the value of $1/a$ that in our opinion are responsible for most of the $\sim 20\%$ differences in $1/a$ between runs with different N . Table 3 gives values of da^{-1}/dt obtained by fitting straight lines to $1/a$ over intervals $1 \leq t - t_h < 4$ and $5 \leq t - t_h < 8$. There is an apparent, though slight, decreasing trend of the hardening rate with N in the former interval, while the latter interval shows no identifiable trend. We also give in Table 3 the dimensionless hardening rate H computed from the measured values of da^{-1}/dt using equation (14); the stellar density and velocity dispersion in that expression were evaluated by averaging inside a sphere of radius $r = 0.04$ around the binary (for evaluating the velocity dispersion, we excluded the center $r < 0.01$ where stars are strongly perturbed by the binary). We find that H ranges from ~ 6 to ~ 9 , consistent with the moderately-hard binary results of three-body scattering experiments summarized above. The lack of a noticeable N -dependence in the hardening rate is consistent with the fact that the central stellar density and velocity dispersion also do not vary substantially with N (Figure 8d,e).

The orbital eccentricity of the BH binary evolves with time as well (Figure 8c). The eccentricity was evaluated from the binding energy and the angular momentum of the binary; before t_h , we corrected the BH masses by adding the mass in stars bound to each of them, $M_\bullet + M_*(r \leq a/2)$. The initial orbital eccentricity of the galaxies is ~ 0.75 and this eccentricity is reflected in the relative orbit of the two BHs when they first fall to the

center. However the orbit rapidly circularizes due to the strong density gradients in the cusp, and by $t = t_h$ the eccentricity is essentially zero. From that point on, e grows at an approximately constant rate, albeit with substantial fluctuations. The fluctuations do not seem to be strongly correlated with N . Final values of e range from ~ 0.15 to ~ 0.3 . Table 3 gives average values of the dimensionless eccentricity growth rate K obtained by fitting the integrated form of equation (16), assuming constant K , to data for $t > 11.6$. In runs A2 and A4, the growth of eccentricity is well approximated by the relation

$$e = K \ln \left(\frac{a_{ecc}}{a} \right) \quad (27)$$

with $K = 0.13$ and $a_{ecc} = 0.001$. (The three-body scattering results of Quinlan (1996) predict a substantially lower rate, $K \approx 0.04$ for $e = 0.3$ and $V_{bin}/\sigma_* = 10.0$, although they allow for $K = 0.13$ when $e \gtrsim 0.4$ and $V_{bin}/\sigma_* \gtrsim 30.0$.) If the eccentricity continued growing at the rate predicted by equation (27), it would reach $e = 0.5$ for $a \approx 2 \times 10^{-5}$, which is near the semimajor axis when the hardening rate due to gravity wave emission becomes larger than that due to stellar ejection (§6). Equation (22) would then imply a gravitational radiation time scale ~ 5 times shorter than if the binary were circular.

Figure 8f shows the mass ejected by the binary M_{ej} as a function of time. We monitored the mass ejection by counting stars with positive energies, or equivalently, with galactic escape velocities. This conservative criterion underestimates the number of stars ejected by a moderately hard binary ($V_{bin}/\sigma_* \lesssim 4$) because some of these stars are not energetic enough to escape the galaxy. Ejected mass at the end of each run is close to M_{12} , the total mass of the binary. We fit $M_{ej}(t)$ to the integrated form of the mass ejection law, equation (18), to derive J and a_{ej} . The fit was satisfactory for $t \gtrsim t_h + 2$; the fitted values of J and a_{ej} are given in Table 3. We find $J \approx 0.5$ which is consistent with the scattering experiments cited above. At

earlier times the mass ejection law with constant J underestimates M_{ej}/M_{12} . As was the case for the hardening rate, the ejected mass does not show a strong dependence on particle number in these simulations.

Readers should not compare the ejected mass in our simulations with that in the simulations of Quinlan & Hernquist (1997); these authors measured greater M_{ej} for a smaller increase in $1/a$, but they also used a different, more liberal definition for M_{ej} . The primary focus of our study is to relate the dynamics of the BH binary's hardening to the *observable* responses of the stellar nucleus, such as the decrease in slope of the central cusp, and the precise definition of M_{ej} is not important for any of the physical conclusions drawn here.

Although the supply of stars to the binary BH remains high throughout our simulations, there is nevertheless a steady drop in the stellar density as stars are ejected from the core. This can be seen, indirectly, in the slight curvature of the hardening rate plot (Figure 8b), and directly in the change in density within a sphere of radius 0.04 centered on the binary (Figure 8d). More detailed information about the change in the stellar mass distribution is given in Figure 7. Most notable is the drop in central density between the first profile, at $t = t_h$, and the second that is offset in time by one N -body unit. This is the same rapid drop in density that was discussed above (§2), associated with formation of the hard binary. The amount of mass ejected during this short time interval is somewhat greater with smaller N , which is also evident in Figure 8d. We attribute this mild N -dependence to statistical fluctuations, and also to a spurious effect associated with the wandering of the BH binary: since density profiles in Figure 7 are centered on the binary, densities at radii less than the wandering radius may be artificially lowered.

There is no suggestion that a “hole” is forming around the BH binary; apparently the supply of stars is great enough that the binary can eject of order its own mass without driving the central density to a very small value. The central density profile, which is slightly steeper than $\rho \sim r^{-1}$ at $t = t_h$, becomes slightly flatter than r^{-1} by the end of the integrations (Figure 7) and would presumably become ever flatter if the integrations were continued to longer times. The inner density profile remains well described as a power law at all times.

4.3. Brownian motion

Figure 9 shows the trajectory of the two BHs at high spatial resolution in our simulations. The Brownian motion is apparent as a sudden change in the character of the BH orbits at $t \approx t_h$. Prior to this time the trajectories are smooth and symmetric, reflecting the dynamical-friction induced coalescence of the two cusps. However starting at $t \approx t_h$ the motion becomes more chaotic, resembling a random walk. Figure 9 shows clearly that the amplitude of the random motion is a decreasing function of N , as expected from equipartition arguments (equation 19). We quantified the N -dependence by computing $\sigma_{12} \equiv \sqrt{\langle v^2 \rangle / 3}$ for the binary's center of mass. This quantity exhibits almost no evolution with time (Table 3); Figure 10 shows averages for $t > t_h$. The equipartition relation $\sigma_{12} \propto M_{12}^{-1/2}$ is approximately satisfied. Of interest is the amplitude of σ_{12} which is expected to be slightly larger for a binary

BH than for a single BH (Merritt, 2001). We were unable to check this prediction by a direct comparison between our two- and single-BH runs since the latter were not extended long enough that an accurate characterization of the BH's Brownian motion could be obtained. Instead, we compared σ_{12} with the velocity dispersion predicted by equation (19), $\sigma_{12} = (m_*/M_{12})^{1/2} \sigma_*$. This requires a choice about how to evaluate σ_* , which depends weakly on radius. In Figure 10 we plot a range of predictions for σ_{12} based on measured values for σ_* within $r = 0.2$ ($\sigma_* \approx 0.6$) and $r = 0.01$ ($\sigma_* \approx 1.0$). The rms velocity of the binary appears to be slightly greater than expected for a point mass, as predicted by Merritt (2001).

Brownian motion of a massive binary was described in a few earlier studies. Quinlan & Hernquist (1997), in a series of N -body simulations, noticed a wandering of their BH binary with an amplitude 5 – 10 times greater than expected on the basis of an equation like (20); they attributed the discrepancy to inelastic scattering of ejected stars. Makino (1997) carried out N -body simulations similar to those of Quinlan & Hernquist (1997) but using a more conservative, direct-summation code and no mass spectrum for the field stars. Makino's Figure 7 shows a wandering amplitude that scales as $\sim N^{-1/2}$, and the rms velocity of the binary appears to be comparable to that expected for a point mass. Makino's results seem consistent with those obtained here and with the predictions of Merritt (2001): a massive binary at the center of a dense cusp should exhibit only slightly greater Brownian motion than a point particle of the same total mass. It is not clear why Quinlan & Hernquist (1997) found a much greater amplitude for the Brownian motion in their simulations; some possible reasons for the discrepancy are discussed in

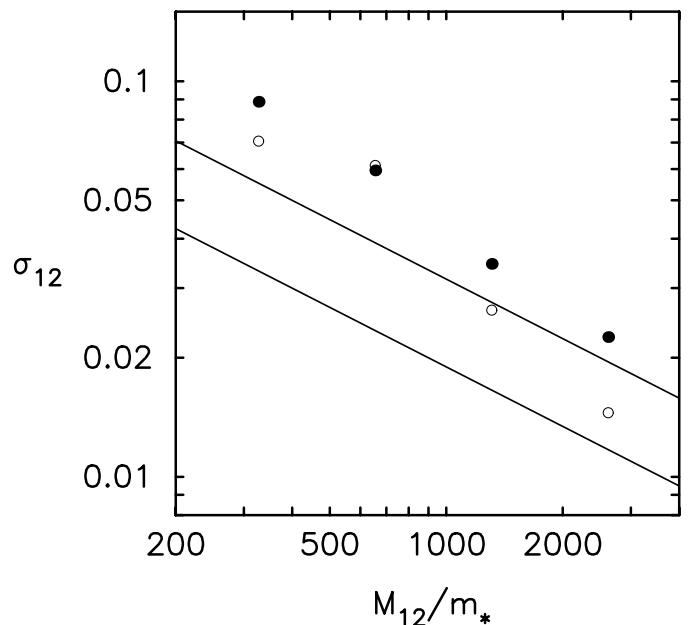


FIG. 10.— 1D rms center-of-mass velocity of the BH binary's Brownian motion, in the merger plane (filled circles) and perpendicular to the merger plane (empty circles). Equipartition of energy implies $\sigma_{12} = (M_{12}/m_*)^{-1/2} \sigma_*$ which depends on the region over which we average σ_* . Solid lines show the band of values consistent with equipartition between $\sigma_* = 0.6$ ($r < 0.2$) and $\sigma_* = 1.0$ ($r < 0.01$), the binary's radius of influence.

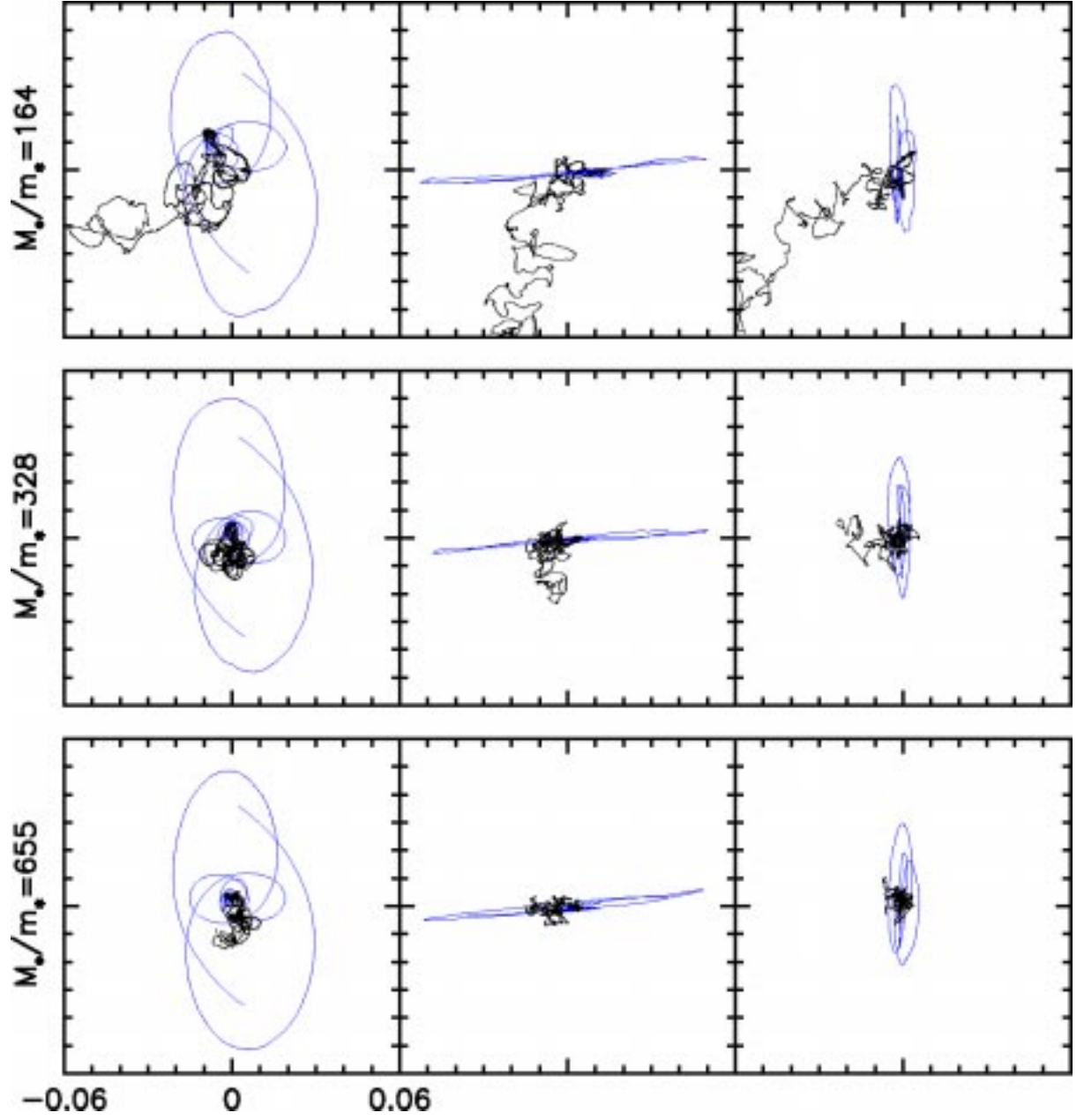


FIG. 9.— Trajectories of the BHs before formation of a hard binary (blue curves) and after (black curves). The character of the motion changes suddenly to a random walk, characteristic of Brownian motion, after the binary forms. The amplitude of the Brownian motion decreases with increasing BH mass (as indicated on the left).

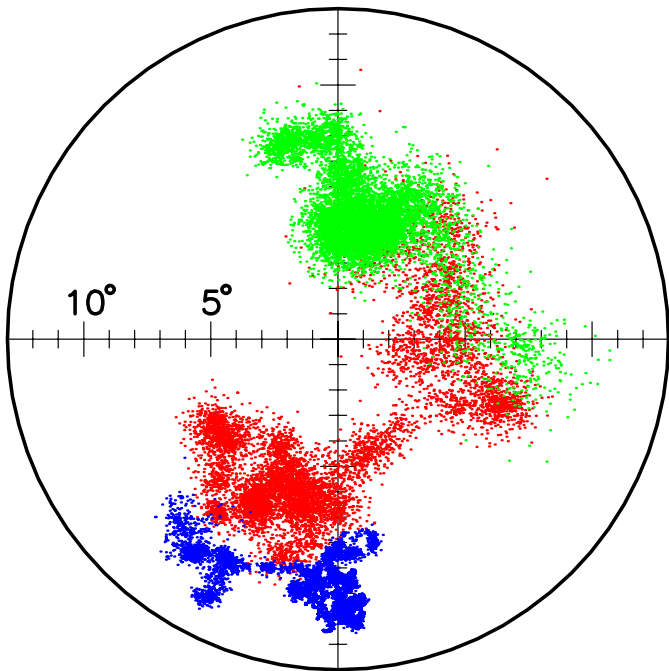


FIG. 11.— Angular inclination of the BH binary’s axis of rotation. Red dots: run A8 ($M_{\bullet}/m_{*} = 164$); green dots: run A4 ($M_{\bullet}/m_{*} = 328$); blue dots: run A2 ($M_{\bullet}/m_{*} = 655$). Binary’s angular momentum unit vector in polar coordinates $\hat{n}(\theta, \phi)$ is shown as a dot at position $(\theta \cos \phi, \theta \sin \phi)$. Center of the plot corresponds to rotation in the merger plane. Inclination of the binary undergoes a random walk with an amplitude that decreases with increasing M_{\bullet}/m_{*} .

Merritt (2001).

The orientation of the binary is also affected by encounters. Figure 11 shows the direction of the binary’s angular momentum vector $\hat{n}(\theta, \phi)$ where each dot represents the angular tilt of \hat{n} from the merger axis. Points shown correspond to $t > t_h$; before the binary is hard, its orientation changes at much higher rate, albeit with similar amplitude, due to transient bulk perturbations occurring during the galactic merger. After t_h , the bulk torques are negligible, but the orientation still changes due to torques imparted by elastic and inelastic encounters with stars. It is evident in Figure 11 that the net effect of stellar ejections is a random walk in the tilt-space (θ, ϕ) and that the rate of orientation-changing decreases both with hardness and with N . We observed a maximum tilt of $\theta \approx 12^\circ$ degrees from the merger axis.

4.4. N -dependence of the evolution

As discussed above, we do not observe an appreciable dependence of the binary hardening rate on N in our simulations. This is reasonable since the expected hardening rate (equation 14) depends only the mean density ρ and velocity dispersion σ_{*} of the field star distribution, not on the masses of field stars.

Some earlier studies noticed a more appreciable N -dependence. Quinlan & Hernquist (1997) found that the decay rate dropped with N until $N \approx 10^5$, then seemed to level off at $N = 2 \times 10^5$. Makino (1997) described the N -dependence of the decay rate as weaker than N^{-1} but gave no further details. In both of these studies, the N -dependence of the hardening rate was attributed to the

Brownian motion, since larger N implies a reduced amplitude of wandering and hence a smaller pool of stars that can interact closely with the binary. Quinlan & Hernquist showed that the binary’s hardening rate dropped rapidly to zero if the binary was artificially fixed in space. This result is also implicit in the study of Zier (2000) who calculated the rate of depletion of stars around a massive binary that was fixed in space.

Why do we fail to see any clear N -dependence of the hardening rate in our simulations – given that the Brownian motion *does* vary in the expected way with N (Figure 10)? The main reason, we believe, is our very different initial conditions, which guarantee a larger supply of stars to the binary than in earlier studies. We noted above that the models of Makino (1997) and Quinlan & Hernquist (1997) had much lower central densities than ours at the time of formation of the hard binary. The supply of stars was correspondingly smaller, implying a more rapid depletion of the “loss cone;” once this occurs, the supply of stars is essentially cut off and any further decay depends on N -dependent processes such as loss-cone refilling and Brownian motion. The origin of the much lower initial density in the Quinlan & Hernquist (1997) simulations may be seen in their Figure 1c, the evolution of the Lagrangian radii for a run where the BHs have a combined mass that is 1% of the galaxy mass, comparable to the value in our simulations. The stellar mass within a sphere of radius 0.01 drops from $\sim 1\%$ initially to $\sim 0.05\%$ by the time of formation of the hard binary. In our simulations, the drop is from $\sim 1\%$ to only $\sim 0.3\%$ over a comparable interval of time (Figure 4). The reason for the much greater density drop in the Quinlan & Hernquist simulations was their choice of initial conditions: the BHs were initially placed far *outside* the central cusp and fell in. Makino’s (1997) models had large cores from the start.

The “supply” of stars in our models can be defined, very approximately, as the number of stars (at $t = t_h$, say) with pericenters less than some critical value $p_{crit} \approx a$. This definition ignores changes in the stellar density profile that result from the changing potential, as well as any back-reaction of the binary’s motion on the stellar distribution. We also ignore any dependence of p_{crit} on stellar velocity, even though low-velocity stars have a larger cross section for interaction with the binary than high-velocity stars. Figure 12 shows M_{crit} , the mass in stars with pericenters below p_{crit} , as a function of p_{crit} . This was computed by counting orbits penetrating, or entirely contained within, the sphere of radius equal to p_{crit} in an $N = 2^{18}$ particle realization of our initial model (equation 3) of pre-merger galaxies. For $p_{crit} \lesssim 0.1$, the function is approximately a power law

$$\frac{M_{crit}}{M} \approx 1.8 \times \left(\frac{p_{crit}}{r_0} \right)^{0.84} \quad (28)$$

where M is the mass of the galaxy and r_0 is the half-mass radius. The figure also shows $1/a_{crit}$, the inverse semimajor axis attainable for an ejected mass of M_{crit} . This was computed from the relation

$$\frac{a_{crit}}{a_{ej}} = e^{M_{crit}/JM_{12}} \quad (29)$$

using the fitted values of J and a_{ej} (Table 3). Taken together, equations (28) and (29) relate the size of the region

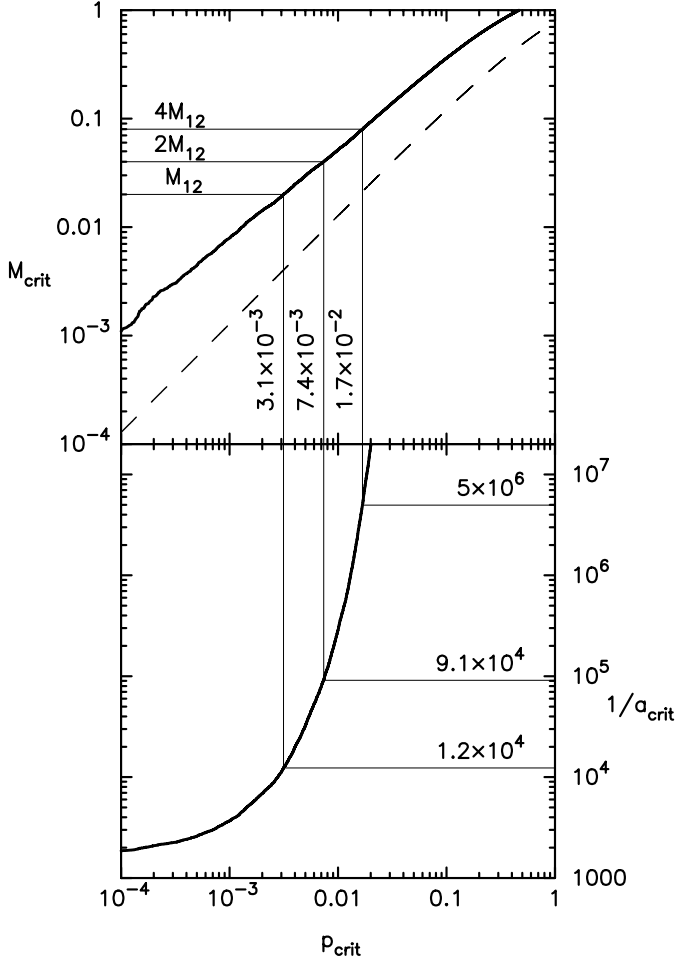


FIG. 12.— Relation between the ejected mass and the shrinking of the BH binary. Upper panel is the mass in stars with pericenter distances smaller than a given value p_{crit} as function of p_{crit} (solid curve). For comparison, we show the mass in stars inside radius $r = p_{crit}$ (dashed curve). M_{crit} is the mass accessible to the BH binary provided that it can visit every point within a distance p_{crit} from the center of the galaxy. Lower panel is the maximum inverse semimajor axis $1/a_{crit}$ at which the binary will stall if it can interact only with stars having pericenter distances less than p_{crit} (solid curve).

accessible to the binary (p_{crit}) to the minimum semimajor axis that the binary can reach by ejecting all stars visiting the region (a_{crit}).

In order for the binary to have access to sufficient mass allowing it to decay to $a^{-1} \approx 10^4$, roughly the final value in our simulations, Figure 12 suggests that it must eject all stars with pericenters lying within ~ 0.003 . This is comfortably *smaller* than the wandering radius even in our largest- N simulation ($r_w \approx 0.01$; Table 3). We believe that this explains why none of our binaries have managed to “deplete the loss cone” and stall – their supply of stars was never depleted, at least over the interval of time represented by the simulations. Combining Figures 10 and 12, we estimate that N would have to be increased to $\sim 5 \times 10^5$ per galaxy in order for the Brownian motion to be small enough that the supply of stars would be exhausted by the time that $1/a = 10^4$.

Given that the “loss cone” is never depleted in our simulations, how great is the effect of two-body relaxation on the decay rate of the binary? We estimated, using stan-

dard expressions like equation (21), that scattering of stars into the loss cone probably does make a significant contribution to the decay rate of the binary in our lowest- N run. For the larger N runs it is probably of negligible importance. This conclusion would presumably change if the loss cone ever became fully depopulated.

What would happen in this case – if the supply of stars to the binary were depleted, either by continuing the integrations to much later times, or by using a larger N and thereby reducing the amplitude of the binary’s Brownian motion? Our guess is that the decay would in fact stall, producing a BH binary whose separation remained nearly constant for extended periods of time. The same would presumably also result from initial conditions with much lower central density, for instance, the merger of two giant elliptical galaxies. We return to the question of the persistence of BH binaries in §6.

We caution that our simple picture, of a BH binary wandering against a fixed distribution of stars, is not completely correct. In fact the binary “carries” the cusp with it to a certain extent. This is clear from the density profiles plot, Figure 7: the cusp continues as a power law to radii much smaller than the wandering radius. It follows that the wandering of the binary is a complex process involving time-dependent changes in the stellar distribution, and these changes probably affect to some extent the supply of stars to the binary in a way not reproduced in our simple analysis. Brownian motion of a massive object in a density cusp would be a fruitful topic for further study.

5. KINEMATICS

Just as the BH binary affects the stellar density profile at distances as large as the break radius, so we expect the presence of BHs to shape the remnant’s kinematical properties well beyond the binary’s gravitational radius of influence. Here we present simulated “observations” of our largest- N merger remnants, from runs A2 and B2, in zero dimensions (circular apertures), one dimension (slit) and two dimensions (integral field). The purpose is to generate predictions that can be tested with the current generation of high-spatial-resolution spectrographs such as STIS, OASIS and SAURON. Following this, we present several revealing views of the remnant in ways that are not directly accessible to astronomical observation.

We “observed” our galaxies in two steps (Appendix C). Starting from the stellar velocities projected into an edge-on view of the galaxy, we recovered the line-of-sight velocity distributions (LOSVDs) non-parametrically via maximum penalized likelihood (MPL) (Merritt, 1997). The MPL estimate $\hat{N}(V)$ of an LOSVD is computed on a grid in velocity such that it maximizes the log-likelihood of the distribution of line-of-sight projected stellar velocities inside an aperture, subject to a penalty function that measures the lack of smoothness of $\hat{N}(V)$. Once $\hat{N}(V)$ was obtained, we expanded it into its Gauss-Hermite (GH) moments defined by Gerhard (1993) according to the prescription of van der Marel & Franx (1993). Of particular interest are the four parameters in the GH expansion (V_0 , σ_0 , h_3 , h_4) that quantify, respectively, the mean velocity and velocity dispersion of the Gaussian prefactor, and the odd and even first-order departures from a Gaussian distribution. (Henceforth in this section, the terms “mean ve-

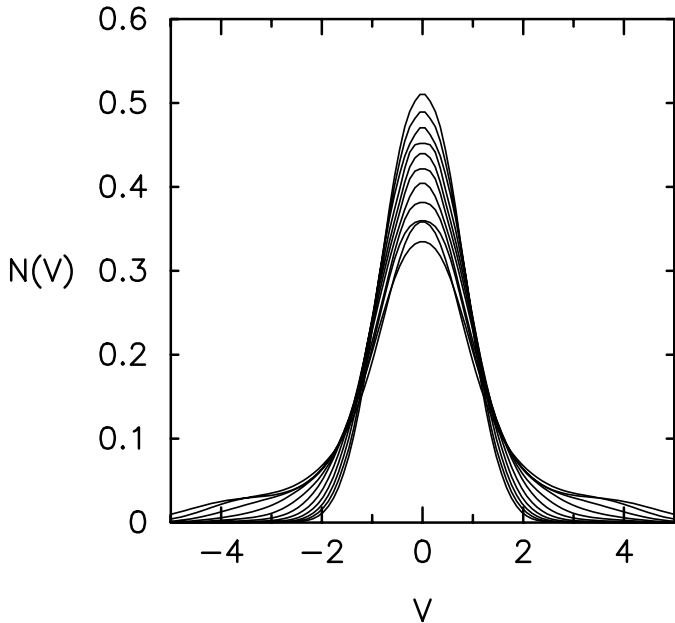


FIG. 13.— Line-of-sight velocity distributions (LOSVDs) of the merger remnant in the run A2 as a function of aperture diameter D . The aperture is circular and centered on the BH binary; its diameter varies from $D = 0.002$ (LOSVD with the shortest peak and the broadest wings) to $D = 0.2$ (LOSVD with the tallest peak and the narrowest wings).

locity” and “velocity dispersion” are used in this restricted sense.) These parameters, however, are insensitive to the power-law wings expected in LOSVDs in the vicinity of a BH (van der Marel, 1994) and hence it is important to consider the full LOSVD.

To increase the resolution inside each circular aperture of diameter D , we superposed one hundred snapshots of the galaxy that were sampled over 1 N -body unit in time, corresponding to ~ 10 crossing times at a radius $r \sim 0.1$ from the center. Averaging over such a wide time interval ensures that stars in the aperture are sampled at random orbital phases. In this procedure each dataset was shifted in space so that the BH binary lay at the center. The centering could cause spurious smoothing on scales smaller than the radius of the Brownian motion of the binary (cf. §4) which however amounts to not more than $r_w \approx 0.0084$ in the run A2 that was used for this purpose. When superposing datasets, the stellar velocities were left in their original frame. Orbits closely bound to the BH binary may follow the binary on its random Brownian trajectory, thereby incurring a net drift in the velocity. This drift, however, scales as $(m_*/M_{12})^{1/2}\sigma_* \approx 0.05$ and can be ignored at this stage.

We anticipate the discussion in §6.1 by quoting typical values for the physical scale of our models. Scaling to a dwarf elliptical galaxy like M32 gives a factor $\mathcal{L} \approx 50$ pc for converting model dimensions into physical lengths. In the case of a giant elliptical galaxy like M87, this factor is ~ 3 kpc. Note that the radius of gravitational influence r_{gr} of the BHs is ~ 0.02 , independent of the scaling.

In Figure 13 we show a family of LOSVDs from the run A2 for a range of diameters D of a circular aperture centered on the BH binary. The LOSVD that is most nearly Gaussian is seen inside an aperture with diameter about

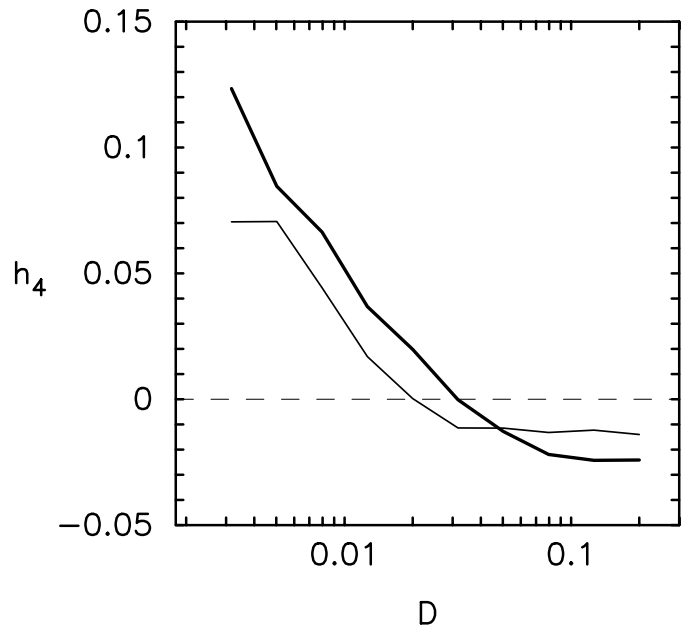


FIG. 14.— Fourth Gauss-Hermite moment h_4 as a function of aperture diameter D . The aperture is centered on the BH binary in the run A2 (thick line) and on the single BH in the run B2 (thin line).

twice the BH binary’s radius of influence, $D \sim 2r_{gr}$. The LOSVD seen in the smallest diameter aperture $D = 0.002$ has the shallowest peak and broadest wings, while the one seen in the largest diameter aperture $D = 0.2$ has the steepest peak and almost non-existent wings. These differences are reflected in the values of h_4 shown in Figure 14. In general, positive h_4 indicates that the LOSVD is sharp (or “triangular”) at the top and decays more mildly on the sides; negative h_4 indicates that the LOSVD is broad (or “boxy”) at the top and steep on the sides, reflecting a sharp maximum velocity cutoff.

We observe that h_4 decreases from $h_4 \approx 0.12$ for $D = 0.003$ to $h_4 \approx -0.024$ for $D = 0.2$ passing through zero for $D \approx 0.03$. A similar trend was derived by van der Marel (1994) under the assumptions of isotropy and spherical symmetry for a model of M87 with a $5 \times 10^9 M_\odot$ BH; the size of the aperture influences only the overall normalization, and not the velocity dependence, of the LOSVD in the large-velocity limit. For comparison, in Figure 14 we also plot h_4 from the run B2 where two BH were combined into one at $t = t_h$. The absolute amplitude of h_4 around the single BH is smaller than around the binary, which is counterintuitive in view of the even wider wings (not shown here) that we found for small apertures in the run with one BH. The Gauss-Hermite moments, however, are sensitive to the velocity profile in the range $V \lesssim 2\sigma_0$ and indifferent to the high-velocity behavior. It is therefore possible that an LOSVD is both “boxy” at low velocities ($h_4 \lesssim 0$) and “wingy” in high velocities (which may or may not imply a positive h_4).

In Figure 15 we plot the major axis slit kinematics of the merger remnant in the run A2 (BH binary) and B2 (single BH). To increase the resolution, we added together six views of the major axis, rotated by angles $k\pi/3$, $k = 0, 1, 2, 3, 4, 5$, around the minor axis, from the original line of sight ($k = 0$). This yields V_0 and h_3 that are odd

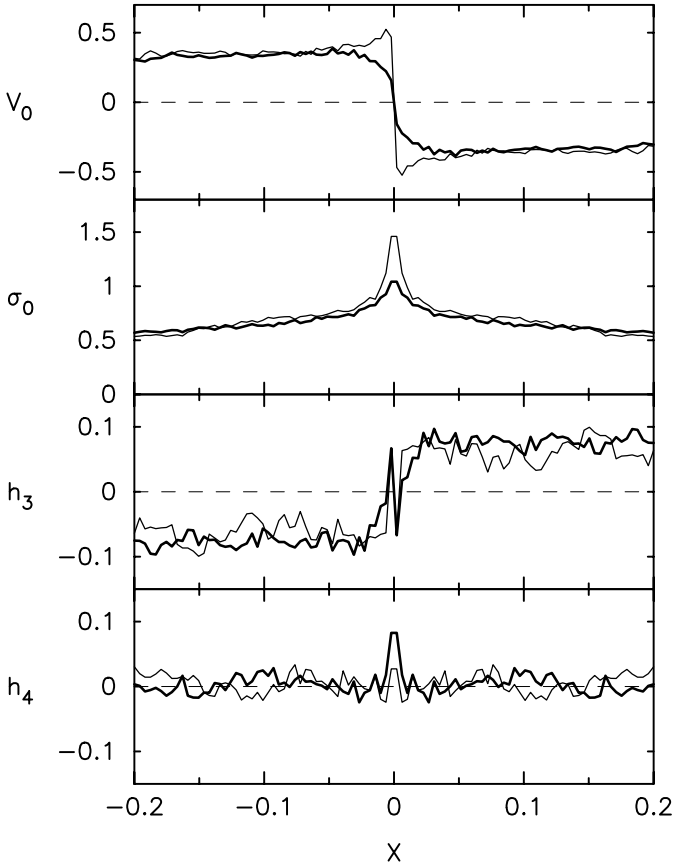


FIG. 15.— Slit kinematics of the run A2 (BH binary; thick line) and B2 (single BH; thin line). Slit is positioned along the major axis and centered on the BH. The aperture diameter (slit width) varies from $D = 0.02$ at $X = \pm 0.2$ to $D = 0.004$ at the center. Parameters V_0 , σ_0 , h_3 and h_4 in the Gauss-Hermite expansions of the LOSVDs were calculated as described in the text.

and σ and h_4 that are even under reflection $X \rightarrow -X$. We first established the position-dependent maximum aperture diameters ($D = 0.004$ at the center and $D = 0.02$ at the ends) and then narrowed the apertures with the requirement that the number of stars inside each aperture remain larger than a fixed amount (4096), ensuring homogeneous statistics across the slit and uncorrelated sampling ($D < \Delta X$) at the center.

We also computed spatial (unprojected) kinematical quantities, in particular, the rotational velocity v_ϕ and three diagonal moments of the velocity dispersion tensor $\sigma_r, \sigma_\phi, \sigma_\theta$. This was done by averaging inside circular wedges of opening angle $|\theta - 90^\circ| < 30^\circ$ (major axis) and $|\theta - 90^\circ| > 30^\circ$ (minor axis). Figure 16 plots spatial kinematical properties of the galaxy immediately following the merger (left column) and of the final galaxy (right column) along the major and minor axes. We also provide kinematical moments of the merger remnant where the two BHs were coalesced into one at $t = t_h$ (dotted line). We will argue below (§6.4) that this model might be a good representation kinematically of a “power-law” elliptical galaxy.

All major elements of the slit kinematics that we observe in the simulations are also found in the kinematical profiles of real galaxies.

1. *Rotation curve.* Our models exhibit flat or slightly

falling rotation curves at the outer radii, $|X| \gtrsim 0.05$, with $V_0/\sigma_0 \approx 0.5$. At these radii, the runs with one and two BHs are essentially identical kinematically. At inner radii, $|X| \lesssim 0.05$, the two rotation curves differ substantially. The rotation curve near the single BH rises and appears to diverge near the center indicating a nearly-Keplerian rotation pattern dominated by the BH’s potential. Near the binary BH, however, the rotation curve drops, with a $\sim 50\%$ smaller V_0 at the knee marking the drop than in the case with a single BH. Our choice of the terms “inner” and “outer” is not arbitrary; in fact, transition between the two regions coincides with the break radius defined above (§3). We therefore suggest that the same physical mechanism responsible for the shallowing of the nuclear density cusp also manages to somehow attenuate the circumnuclear rotation.

We propose a mechanism closely related to mass ejection by a BH binary (cf. §4) that leads to precisely this effect. When two stars pass near the binary in opposite directions, the star on a co-rotating (prograde) orbit is more likely to be captured by one of the BHs since it can interact with the BH over a larger orbital phase than the counter-rotating (retrograde) star. As co-rotating stars are preferentially ejected from the nucleus, there will be an increase in the relative number of counter-rotating stars, thereby attenuating the net rotation. In our simulations this effect is too small to reverse the direction of rotation in the nucleus but it plausibly explains the 50% difference between the runs with one and two BHs. This interpretation implies a concrete physical prediction: rotation curves in galaxies with shallow central density cusps (or “core” galaxies, cf. §6) should turn over near the break radius and exhibit systematically lower rotation to within several dynamical radii of the BH than those in galaxies with steep density cusps (or “power law” galaxies).

The rotation curve of our single-BH (“power-law”) model looks similar to that of M32 as observed with STIS on HST (Joseph et al., 2000).

2. *Velocity dispersions.* The central velocity dispersion exhibits a sudden upturn at a distance $\sim r_{gr} \approx 0.02$ from the BHs. The spike is more pronounced in the run with one BH ($\sigma_{0,max} = 1.46$) than in the run with two ($\sigma_{0,max} = 1.04$). This difference is reduced when the “corrected” velocity dispersion $\sigma \equiv \sigma_0(1 + \sqrt{6}h_4)$ (not shown) is used; σ is a closer approximation than σ_0 to the true rms velocity (van der Marel & Franx, 1993). In the run with one BH, $\sigma_{max} = 1.56$, while with two BHs $\sigma_{max} = 1.26$. Nevertheless, in both the uncorrected and corrected velocity dispersions, we note a systematically lower value around the binary BH compared with the single BH out to a radius of ~ 0.1 . Lower dispersions around the BH binary may at first sight strike the reader as unexpected, given that the binary injects energy into the system as it hardens. But the loss cone is populated largely by radial orbits; as the binary captures, ejects, and removes radial orbits from the nucleus, the radial dispersion σ_r drops while the tangential dispersion $\sigma_t \equiv \sqrt{\sigma_\phi^2 + \sigma_\theta^2}$ remains constant, resulting in a decrease in the average dispersion $\sigma = \sqrt{\sigma_r^2/3 + 2\sigma_t^2/3}$.

We in fact observe tangentially-anisotropic motions out to radii $r \lesssim 0.03$, as shown in Figure 16. In the bottom right panel we show the variation of the anisotropy pa-

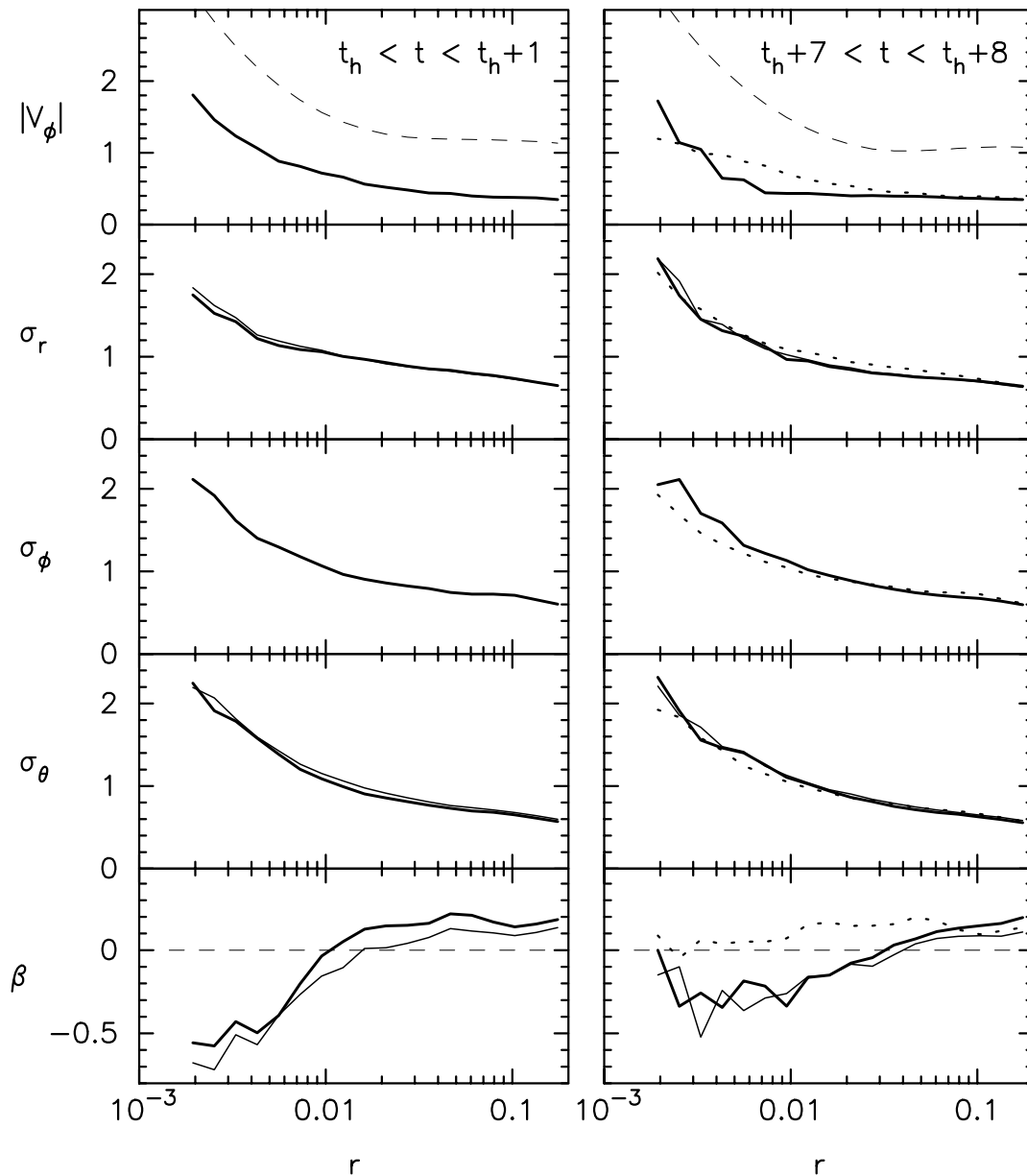


FIG. 16.— Internal kinematics along the major axis (thick curve) and minor axis (thin curve) immediately after the formation of a hard binary (left column) and at the end of simulation (right column). Solid curves: run A2 with a BH binary. Dotted curves: run B2 with single BH (major axis only). The major and minor axis moments were computed by direct averaging inside the circular wedges $|\theta - 90^\circ| < 30^\circ$ and $|\theta - 90^\circ| > 30^\circ$, respectively. First row: rotational velocity v_ϕ ; thin dashed curve is the circular velocity $v_c \equiv \sqrt{G(M_{12} + M(r))/r}$. Second to fourth rows: moments of the rms velocity dispersion (σ_r , σ_ϕ , σ_θ). Fifth row: anisotropy parameter $\beta \equiv 1 - \sigma_t^2/\sigma_r^2$ where $\sigma_t^2 \equiv (\sigma_\phi^2 + \sigma_\theta^2)/2$. The tangentially-anisotropic central region ($\beta < 0$) grows with time, from $r \approx 0.01$ at $t = t_h$ to $r \approx 0.03$ at $t = t_h + 7$. The simulation with a single BH exhibits no appreciable anisotropy.

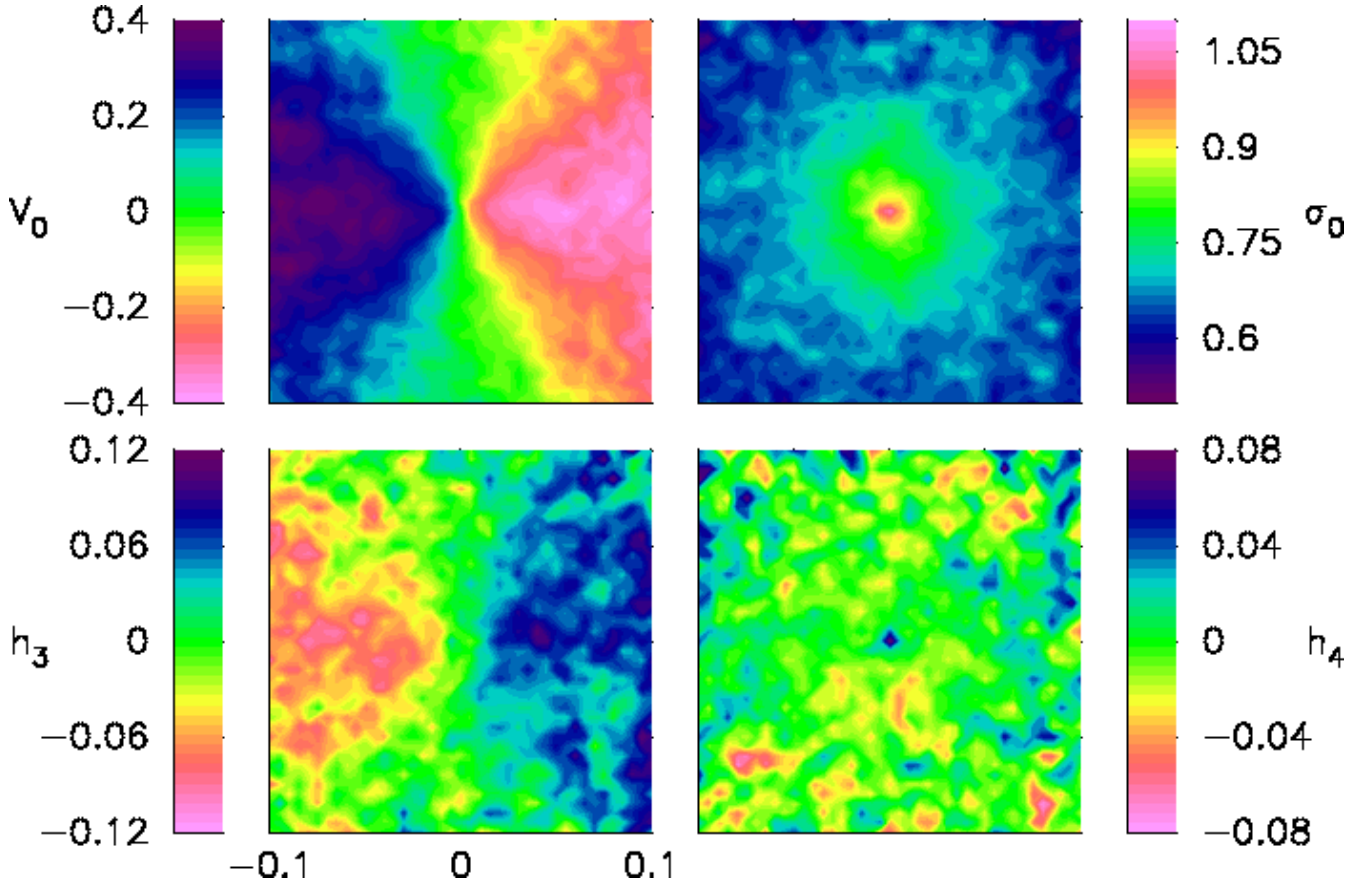


FIG. 17.— Two-dimensional kinematical maps of the merger remnant from run A2. The line of sight is parallel to the plane of the merger; the latter projects into the horizontal, or major, axis of the image. Gauss-Hermite parameters (V_0, σ_0, h_3, h_4) were recovered as described in the text (§5). To increase the resolution inside each aperture, 100 snapshots from $18.1 \leq t \leq 19.1$ were superposed; this interval amounts to $\gtrsim 10$ crossing times at radii $r \lesssim 0.1$ from the center.

parameter β with radius in the final models; β is defined as $\beta \equiv 1 - \sigma_t^2/\sigma_r^2$. In the case of a single BH, $0 \leq \beta \leq 0.2$ at all radii, which we believe is consistent with $\beta \sim 0$. In contrast, in the binary BH model, $-0.4 \leq \beta \leq 0$ for $r < 0.03$ and $0 \leq \beta \leq 0.2$ for $r > 0.03$. The anisotropy that we measure outside the binary’s radius of influence ($r_{gr} \approx 0.01$) is consistent with that from the equivalent run of Quinlan & Hernquist (1997). These authors detected $\beta \approx -1.0$ but only at radii so close to the center that the calculation of β may depend on the resolution effects and the binary’s Brownian motion. Zier (2000), whose binary was fixed in space, found even stronger central anisotropy. We reiterate the prediction of Quinlan & Hernquist (1997) that weak density cusps formed by the action of BH binaries are tangentially anisotropic. We however disagree with the characterization of the anisotropy as “strong;” in fact it is mild ($\beta \lesssim 0.5$) on scales that can be resolved observationally. On smaller scales, we warn against the possibility of contamination of any inferred anisotropy by stars that have recently been ejected or that are interacting strongly with the binary.

3. *Third Gauss-Hermite moments.* h_3 is constant and has a sign opposite to that of the velocity parameter V_0 . If h_3 and V_0 have the same sign, the prograde wing of the LOSVD is wider and the retrograde wing is steeper; if the signs are opposite, the reverse is true. Figure 15 shows h_3 in the runs with one and two BHs; in the for-

mer $|h_3| \approx 0.063$, in the latter $|h_3| \approx 0.074$. We also note the average ratios $\langle h_3 \rangle / \langle V_0 / \sigma_0 \rangle \approx -0.12$ for one BH and $\langle h_3 \rangle / \langle V_0 / \sigma_0 \rangle \approx -0.15$ for two. These ratios are in excellent agreement with the empirical $h_3 - \sigma/V$ relation of Bender et al. (1994) in a sample of 44 elliptical galaxies. All galaxies in the Bender et al. sample show opposite signs of h_3 and V_0 and fall near the relation $\langle h_3 \rangle \approx -0.12 \langle V_0 / \sigma_0 \rangle$. The sudden sign change in h_3 very near the center is a consequence of averaging over apertures; similar features can be seen in the models of Dehnen (1995) and Qian et al. (1995).

Interestingly, our simulations present a direct counterexample to the results of Burkert & Naab (2001) whose simulated mergers always yielded $h_3 / (V_0 / \sigma_0) > 0$ in apparent disagreement with the observations; these authors argue that the presence of disk-like subcomponents may be necessary to reproduce the correct sign of $h_3 / (V_0 / \sigma_0) < 0$. The bulge initial conditions of Burkert & Naab (2001) had $\rho \sim r^{-1}$ central cusps and were thus significantly less concentrated than ours. New simulations of mergers with a range of density profiles are needed to clarify the dependence of h_3 on initial conditions.

4. *Fourth Gauss-Hermite moments.* h_4 is very small except at the very center. At radii greater than twice the BH radius of influence r_{gr} , our data are consistent with $h_4 = 0$. For $r_{gr} \lesssim r \lesssim 2r_{gr}$, h_4 dips into negative values; this is again a consequence of averaging over finite

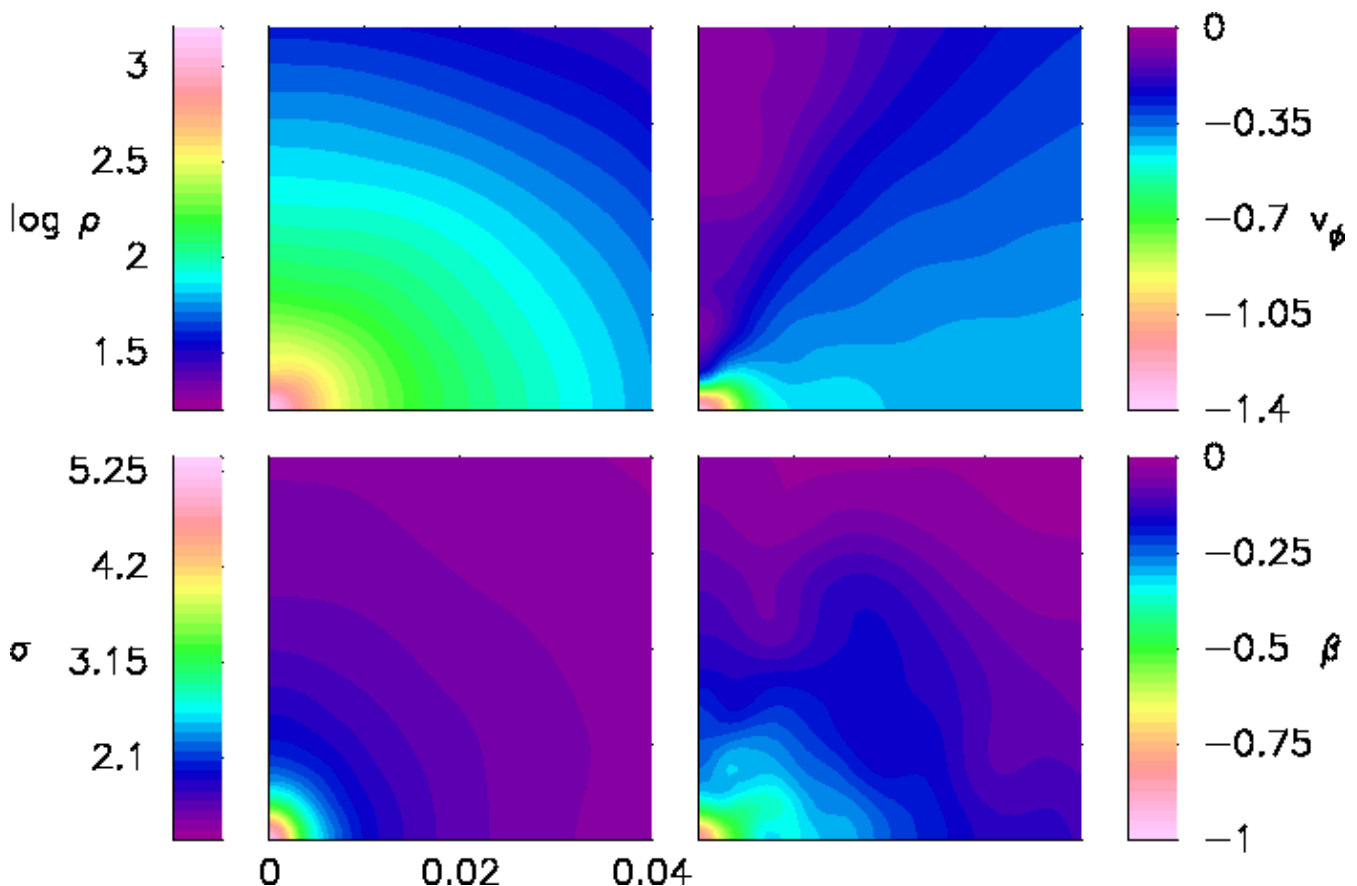


FIG. 18.— Structure of the merger remnant in the meridional plane. The BH binary is at the lower left corner of each panel. Plots are averages over multiple snapshots as described for Figure 5. Horizontal axis is distance ϖ from the Z axis; vertical axis is distance Z from the equatorial plane. ρ is the density of stars, v_ϕ is the average velocity through the meridional plane, $\sigma = \sqrt{\sigma_r^2/3 + 2\sigma_t^2/3}$ is the average 1D velocity dispersion, and $\beta \equiv 1 - \sigma_t^2/\sigma_r^2$ is the anisotropy parameter, where σ_r and σ_t are, respectively, the radial and the tangential velocity dispersions.

apertures (Dehnen, 1995; Qian et al., 1995). For $r \lesssim r_{gr}$, h_4 becomes positive again with peak values of about 0.027 (one BH) and 0.083 (two BHs). Although it is tempting to ascribe this difference to the differing effects of single and dual BHs, the Poisson uncertainties in our determinations of h_4 are large enough that we are reluctant to infer a significant difference between the two models.

The new generation of integral field spectrographs like SAURON (Peletier et al., 2001) can obtain two-dimensional maps of the stellar kinematics from absorption-line spectra at resolutions of less than an arcsecond, corresponding to $\lesssim 1$ pc in nearby galaxies. In Figure 17, we show 2D maps of the parameters V_0 , σ_0 , h_3 and h_4 similar to the maps obtainable with SAURON. The line of sight is parallel to the plane of the merger, which projects into the horizontal, X -axis of the image. To generate these images, we combined 100 snapshots in the time interval $7.5 < t < 8.5$. At every point on an 41×41 grid we started with an aperture of diameter $D = 0.01$ that was then shrunk to smaller D under the condition that the number of stars inside the aperture always remain sufficient for the recovery of the LOSVDs (a few $\times 10^3$).

The rotation pattern seen in projection is symmetric and co-aligned with the merger’s initial orbital axis (vertical axis in the image). The rotation reaches a maximum of about $|V_0| \approx 0.4$ at $r \approx 0.04$ on the major axis before

dropping slightly at larger radii. The velocity dispersion σ_0 exhibits no deviation from a circular pattern in spite of the noticeably elliptical isophotes of the density (Figure 5). The third Gauss-Hermite moment h_3 is largest along the major axis ($h_3 \sim \pm 0.1$ at $R = 0.05$ is typical) and gradually decreases to zero toward the minor axis. Its map resembles the rotation pattern of V_0 , except that the sign of h_3 is opposite from V_0 as noted above. The fourth Gauss-Hermite moment h_4 is consistent with zero everywhere except for the center; however there is a hint of positive h_4 along the major axis and negative h_4 along the minor axis.

In the run B2 where the BH binary was replaced by single BH at $t = t_h$ (not shown here), the rotation field peaks at $V_0 \approx \pm 0.55$ much closer to the BH at $|X| < 0.01$ and remains high $|V_0| \gtrsim 0.5$ in a disk-like circumnuclear region out to $|X| \sim 0.025$.

Finally, in Figure 18 we present 2D maps of ρ , v_ϕ , σ and β in the meridional plane, which is perpendicular to the merger plane and contains the axis of rotation of the merger remnant. Here $\sigma \equiv \sqrt{\sigma_r^2/3 + 2\sigma_t^2/3}$ is the average 1D velocity dispersion and $\beta \equiv 1 - \sigma_t^2/\sigma_r^2$ is the anisotropy parameter. In each panel, the pixels are mapped in coordinates (ϖ, Z) , where ϖ is the distance from the axis of rotation and Z is the distance above the equatorial plane (i.e. the plane of the merger). We found symmetry in all

of the above variables under reflection $Z \rightarrow -Z$, therefore only positive values of Z are plotted. As in Figure 17, the images were generated from a superposition of 100 snapshots in the interval $7.5 < t < 8.5$, but this time we calculated non-parametric kernel estimates for all quantities as described in Appendix B.

6. DISCUSSION

6.1. Scaling

The scaling of our models to physical units will depend on which time step we choose to compare with real galaxies and on which galaxy we choose for comparison. Fortunately there are two quantities that remain nearly or exactly constant with respect to time in our simulations and which are convenient for scaling: the total mass M_{12} of the BH binary, and the velocity dispersion σ_* of stars in the nucleus, outside the region where the stellar motions are strongly affected by the BHs. The first quantity is precisely constant, while the second varies only slightly with time (e.g. Figure 8e) and position (Figure 16).

Furthermore there is a tight relation between BH mass and stellar velocity dispersion in real galaxies, the $M_\bullet - \sigma$ relation, which allows us to reduce the scaling to a single number. For our purposes, the most useful form of the $M_\bullet - \sigma$ relation is

$$M_\bullet = 1.30(\pm 0.36) \times 10^8 M_\odot \left(\frac{\sigma_c}{200 \text{ km s}^{-1}} \right)^{4.72(\pm 0.36)} \quad (30)$$

(Merritt & Ferrarese, 2001a). Here σ_c is the projected velocity dispersion measured in an aperture of radius $r_e/8$ centered on the BH, with r_e the half-light radius of the stars. At ground-based resolutions, σ_c is essentially unaffected by the presence of the BH and measures the velocity dispersion defined by the stellar spheroid. If we equate σ_c with σ_* in our definition for a_h , the semi-major axis of a hard binary (equation 4), we find

$$a_h \approx 1.51 \text{ pc} \left(\frac{M_\bullet}{10^8 M_\odot} \right)^{0.576} \quad (31)$$

In our simulations, $\sigma_*(r_e/8)$ is close to 0.8 in model units at all times after formation of the hard BH binary. The mass of the BH binary is 0.02 in model units; we identify this with M_\bullet , the mass of the BH in the observed galaxy (or the combined mass of the two BHs in the case of a binary). If we define scaling factors $\{\mathcal{M}, \mathcal{V}, \mathcal{L}, \mathcal{T}\}$ for our models, such that the mass in physical units is \mathcal{M} times the mass in model units and similarly for velocity, length and time, then (30) implies

$$0.02\mathcal{M} = 1.30 \times 10^8 M_\odot \left(\frac{0.8\mathcal{V}}{200 \text{ km s}^{-1}} \right)^{4.72} \quad (32)$$

or

$$\frac{\mathcal{M}}{10^9 M_\odot} = 2.27 \left(\frac{\mathcal{V}}{200 \text{ km s}^{-1}} \right)^{4.72} \quad (33)$$

and

$$\mathcal{M} = 50 M_\bullet. \quad (34)$$

The length and time scaling factors are

$$\begin{aligned} \mathcal{L} &= 390 \text{ pc} \left(\frac{M_\bullet}{10^8 M_\odot} \right)^{0.58}, \\ \mathcal{T} &= 1.62 \times 10^6 \text{ yr} \left(\frac{M_\bullet}{10^8 M_\odot} \right)^{0.37}. \end{aligned} \quad (35)$$

We note that these scaling factors are independent of quantities like the break radius r_b . This is appropriate, since the empirical $M_\bullet - \sigma$ relation is also (apparently) r_b -independent, and in our simulations, σ_* (and hence \mathcal{V}) are hardly affected by the creation of a core.

We consider two representative examples for the scaling. Suppose we identify our simulations at early times (before cusp destruction) with a galaxy like M32, a dwarf elliptical with a steep cusp. The BH mass in M32 is $M_\bullet \approx 3 \times 10^6 M_\odot$ (Joseph et al., 2000), giving

$$\begin{aligned} \mathcal{M} &= 1.5 \times 10^8 M_\odot, \quad \mathcal{V} = 110 \text{ km s}^{-1}, \\ \mathcal{L} &= 51 \text{ pc}, \quad \mathcal{T} = 4.4 \times 10^5 \text{ yr}. \end{aligned} \quad (36)$$

The elapsed time in our simulations from $t = t_h$, the time of formation of the hard binary, until the final time step at $t \approx 20$ then corresponds to $\Delta t \approx 4.0 \times 10^6 \text{ yr}$. The final value of a , the semimajor axis of the binary, is $\sim 5 \times 10^{-3} \text{ pc}$ or $\sim 0.04 a_h$, and the final break radius is $\sim 3 \text{ pc}$.

Or we could identify our simulations at late times (after cusp destruction) with a galaxy like M87, a bright elliptical with a weak cusp. The BH mass in M87 is $M_\bullet \approx 3 \times 10^9 M_\odot$ (Macchetto et al., 1997), which gives

$$\begin{aligned} \mathcal{M} &= 1.50 \times 10^{11} M_\odot, \quad \mathcal{V} = 490 \text{ km s}^{-1}, \\ \mathcal{L} &= 2.8 \text{ kpc}, \quad \mathcal{T} = 5.7 \times 10^6 \text{ yr}. \end{aligned} \quad (37)$$

The corresponding elapsed time is $\sim 5.0 \times 10^7 \text{ yr}$, the final value of a is $\sim 0.28 \text{ pc}$ (also $\sim 0.04 a_h$), and the final break radius is $\sim 170 \text{ pc}$. (This value is ~ 3 times smaller than the break radius reported by Lauer et al. (1992), which probably means that M87 has undergone more than one major merger since the era of formation of the supermassive BHs; see §6.3.)

Henceforth we will refer to these as the “M32” and “M87” scalings respectively.

6.2. Cusps and Cores

Our simulations demonstrate that the merger of two galaxies with steep, power-law density cusps ($\rho \sim r^{-2}$) can produce a galaxy with a shallow power-law cusp ($\rho \sim r^{-1}$) inside of a break radius r_b ; the necessary ingredient for the transformation is energy input from a pair of massive BHs. Omitting the BHs (Barnes, 1999), or artificially coalescing them immediately after the merger (§3), preserves the steep cusp. As discussed above (§3), most of the evolution of the central density profile in our simulations takes place during a brief period when the two BHs first form a hard binary. Subsequent ejection of stars by the BH binary produces a gradual flattening of the inner slope (Figure 7). The space density profile $\rho(r)$ is always well described as a power law, $\rho \sim r^{-\gamma}$, at small radii. However the projected density $\Sigma(R)$ looks qualitatively different: a “core” appears, characterized by a log-log slope that falls to zero

at the center (Figure 7). This is natural, since an r^{-1} cusp in space density projects to a logarithmic core in surface brightness (e.g. Dehnen (1993)); only power laws steeper than r^{-1} remain power laws on projection.

Figure 7 invites comparison with the luminosity profiles of real galaxies. These are well known to fall into two classes, the “power laws” and the “cores,” the latter having well-defined break radii. It was initially argued (Lauer et al., 1995) that core profiles were qualitatively distinct from power-law profiles, but Merritt & Fridman (1996) pointed out the effect of projection on a weak power-law cusp and used nonparametric deprojection techniques to verify a power-law dependence of ρ on r even in the core galaxies. More recent work (Gebhardt et al., 1996; Rest et al., 2001) has verified this result in larger samples. Our simulations demonstrate that nuclear density profiles with a range of power-law slopes can be generated in a natural way starting from galaxies with steep cusps, and that the resultant nuclei look like classical cores in projection when the index of the power law is small.

Surface brightness data for elliptical galaxies and bulges are commonly fit to a parametric model, the “Nuker” law:

$$\Sigma(R) = \Sigma_0 \xi^{-\Gamma} (1 + \xi^\alpha)^{(\Gamma-\beta)/\alpha}, \quad \xi = \frac{R}{R_0} \quad (38)$$

(Lauer et al., 1995; Byun et al., 1996). This functional form (and the one due to Ferrarese et al. (1994) on which it was based) has a built-in power-law dependence of Σ on R at small radii. Our simulations preserve a power law in the *space* density at small radii but not the *surface* density. We suggest fitting power-law models like (38) to the *deprojected* density profiles of galaxies, rather than to their surface brightness profiles – or equivalently, fitting the projection of an expression like equation (38) to the surface brightness data. If “core” galaxies really are characterized by weak inner power laws in ρ , as in our models, the fit to the data should thereby be improved.

6.3. The $M_\bullet - r_b$ and $M_\bullet - M_{ej}$ Relations

Part of the motivation for putting core and power-law galaxies into distinct categories was the observation that core galaxies are systematically more luminous than power-law galaxies. Core galaxies have $-24 \lesssim M_V \lesssim -20$ while power-law galaxies are mostly fainter than $M_V = -20$ (Ferrarese et al., 1994; Gebhardt et al., 1996; Faber et al., 1997), although with considerable overlap at intermediate luminosities, $-22 \lesssim M_V \lesssim -20.5$. Recent studies (e.g. Carollo & Stiavelli (1998); Rest et al. (2001); Ravindranath et al. (2001)) have confirmed this systematic difference while weakening the case for a dichotomy; the variation of cusp slope with galaxy luminosity is essentially continuous in the larger samples now available.

What does our model predict? Bright galaxies should have experienced more mergers than faint galaxies and suffered more from the scouring action of binary BHs. This is the reasoning that led Ebisuzaki, Makino & Okumura (1991) to suggest that the “cores” of giant ellipticals (which they took to be regions of constant density, not weak power-law cusps) are generated by binary BHs. Faber et al. (1997) showed that the “core masses” of bright ellipticals scale with galaxy luminosity in roughly the same way as in the simulations of Quinlan & Hernquist (1997).

A point not made by these authors is that a pair of BHs will eject of order its combined mass during *each* merger. The total mass ejected depends both on the final BH mass, and on the number of stages in the merger hierarchy that have occurred since the BHs first formed. If the masses of the BHs at some stage in the merger hierarchy is M_\bullet/n , with M_\bullet the final BH mass, the mass ejected by all the progenitors at this stage is of order $n \times M_\bullet/n \approx M_\bullet$, and the total mass ejected in the complete set of mergers scales both with M_\bullet and with the number of mergers. Since the latter is bigger for bigger galaxies, we expect to see a steeper-than-linear relation between core mass and BH mass in elliptical galaxies.

To sharpen this argument and test it against real galaxies, we need a working definition of “core mass,” or more precisely, for the mass deficit – the mass ejected by the BHs. We define this as the mass needed to bring an observed density profile to a $\rho \sim r^{-2}$ dependence near the center. Here we are assuming, as above, that r^{-2} cusps were universally present before the binary BHs began to do their damage and that they would have been preserved in the absence of BHs. This assumption (similar to the one made in §1 when justifying our choice of initial conditions) is reasonable since: (1) the growth of single BHs in pre-existing cores produces $\rho \sim r^{-2}$ cusps; (2) steep cusps are preserved during mergers in the absence of energy input from supermassive BHs (§3); (3) without BHs, pre-existing cores evolve into something like steep cusps through successive mergers (Makino & Ebisuzaki, 1996); (4) faint elliptical galaxies universally have steep cusps.

Consider then a pair of galaxies with $\rho \sim r^{-2}$ central density cusps which merge to form a galaxy with a shallower cusp, $\rho \sim r^{-\gamma}$, $\gamma < 2$, inside of a break radius r_b . Assume that the density profile of the merger remnant was homologous with that of the merging galaxies before the BHs began to heat the stars. The mass initially within r_b was

$$4\pi \int_0^{r_b} dr r^2 \frac{\sigma_*^2}{2\pi G r^2} = \frac{2\sigma_*^2 r_b}{G} \quad (39)$$

and after mass ejection,

$$4\pi \int_0^{r_b} dr r^2 \frac{\sigma_*^2}{2\pi G r_b^{2-\gamma} r^\gamma} = \frac{2}{3-\gamma} \frac{\sigma_*^2 r_b}{G}. \quad (40)$$

We ignore changes in σ_* (cf. Figure 8). The ejected mass is therefore

$$M_{ej} \approx \frac{2(2-\gamma)}{3-\gamma} \frac{\sigma_*^2 r_b}{G}. \quad (41)$$

The break radius r_b that appears in equation (41) refers to the space density $\rho(r)$, while published break radii R_b are derived from surface brightness profiles $\Sigma(R)$. However the definitions of both r_b and R_b are to an extent arbitrary (e.g. Faber et al. (1997)) especially since we ignore in our definition of M_{ej} the precise form of the density profile near the break radius; furthermore we find that $r_b \approx R_b$ within the uncertainties in our N -body models (Table 3). We therefore feel justified in replacing r_b by R_b in equation (41). We find that M_{ej} defined in this way is larger by a factor ~ 2 than the ejected masses that we measured in our simulations by counting stars that completely escape the galaxy (Figure 8f). There is a simple explanation for this discrepancy: after the ejection of one BH-binary-mass in

stars, the central gravitational pull on the remaining stars will decrease by roughly a half; these stars will then shift to wider orbits thereby increasing the apparent M_{ej} . But this factor of ~ 2 is comparable to other uncertainties in the definition of M_{ej} and we will neglect it in what follows.

Faber et al. (1997) list 16 “core” galaxies for which there are published measurements of γ , R_b and σ_* . Essentially none of these galaxies has an accurately-determined BH mass, but we can use the $M_\bullet - \sigma$ relation (30) combined with measured values of σ_* to estimate M_\bullet . Figure 19 shows the result. The correlation of M_{ej} with M_\bullet is reasonably tight, and the slope of the relation is significantly greater than one, as predicted. By contrast, R_b and M_\bullet are essentially uncorrelated. The brightest core galaxies ($M_V \approx -24$) have $\langle M_{ej} \rangle \approx 10M_\bullet$, while the faintest core galaxies ($M_V \approx -20$) have $M_{ej} \approx M_\bullet$. Ejection of $\sim 10M_\bullet$ in stars in the biggest galaxies seems reasonable since these galaxies are believed to have experienced several mergers since the quasar epoch (Kauffmann, Charlot & Balogh, 2001).

While this success is encouraging, we point out that there are other factors that might contribute to the trend of increasing M_{ej}/M_\bullet with galaxy mass. The “power-law” galaxies have $\gamma \approx 2$ and hence $M_{ej} \approx 0$ according to equation (41). These galaxies would fall far below and to the

left of the “core” galaxies in Figure 19 and so it is reasonable to expect a steeper-than-unit slope for the brighter galaxies that are plotted there. We return in the next section to the question of how the power-law galaxies managed to maintain steep cusps in the presence of mergers.

6.4. Persistence of Steep Cusps in Faint Galaxies

The story just outlined can not be complete, since steep power-law cusps persist in elliptical galaxies as bright as $M_V \approx -22$ (Gebhardt et al., 1996), and are universally present in galaxies fainter than $M_V \approx -20$. How have the cusps in the power-law galaxies managed to avoid destruction by merging BHs? We discuss several possibilities.

1. *Power-law galaxies do not contain supermassive BHs.* While the presence of supermassive compact objects has only been reliably established in a handful of galaxies (see discussion in Ferrarese & Merritt (2000)), several of the best cases are in stellar systems with steep cusps (e.g. the bulge of the Milky Way; M32).

2. *Power-law galaxies were not formed via mergers.* This would contradict standard models for hierarchical structure formation (Lacey & Cole, 1993; Haehnelt & Kauffmann, 2000; Menou, Haiman & Narayanan, 2001), although it is possible that most power-law galaxies have not experienced major mergers since the era when BHs gained most of their mass; we return to this idea below.

3. *Cusps in power-law galaxies are regenerated following mergers.* Faber et al. (1997) suggested that steep cusps might be produced by star formation from fresh gas supplied during mergers. While mergers certainly lead to enhanced star formation, we consider this explanation unlikely since nuclei formed from infalling gas do not resemble featureless power laws (e.g. Mihos & Hernquist (1994)). The scale-free nature of the cusps suggests to us a gravitational origin.

4. *A mechanism exists for extracting energy from BH binaries before they can heat the surrounding stars.* This suggestion is motivated by the observation that the central density profiles in our simulations remain homologous with the initial profile, $\rho \sim r^{-2}$, for a short time after the merger (Figure 3). If some process were effective at extracting the binary’s energy at this stage, at a rate higher than the hardening rate due to stellar ejection, cusp disruption could be avoided, producing a coalesced BH binary in a steep cusp. We tested this idea in a limited way above (§3) by artificially combining the two BHs immediately after the merger; the profile retained its steep power-law character thereafter.

We propose that explanation (4) is the correct one and that the mechanism which extracts energy from the BH binary is gas dynamical in origin. The effects of gas on the evolution of BH binaries have been discussed by a number of authors. Gas may accrete onto the larger of the BHs causing orbital contraction at a rate $\sim M_{12}/\dot{M}$ (Begelman, Blandford & Rees, 1980; Valtonen, 1996). Interaction of the binary with an accretion disk will transfer angular momentum from the BHs to the gas causing the binary orbit to decay (Lin & Papaloizou, 1979a,b; Syer & Clarke, 1995; Artymowicz & Lubow, 1996); the rate is again of order the gas accretion rate (Ivanov, Papaloizou & Polnarev, 1999). Discrete gas clouds, like those near the center of the Milky Way (Coil & Ho, 1999), might also affect the evolution of the binary, particularly when its separation is large. Gas

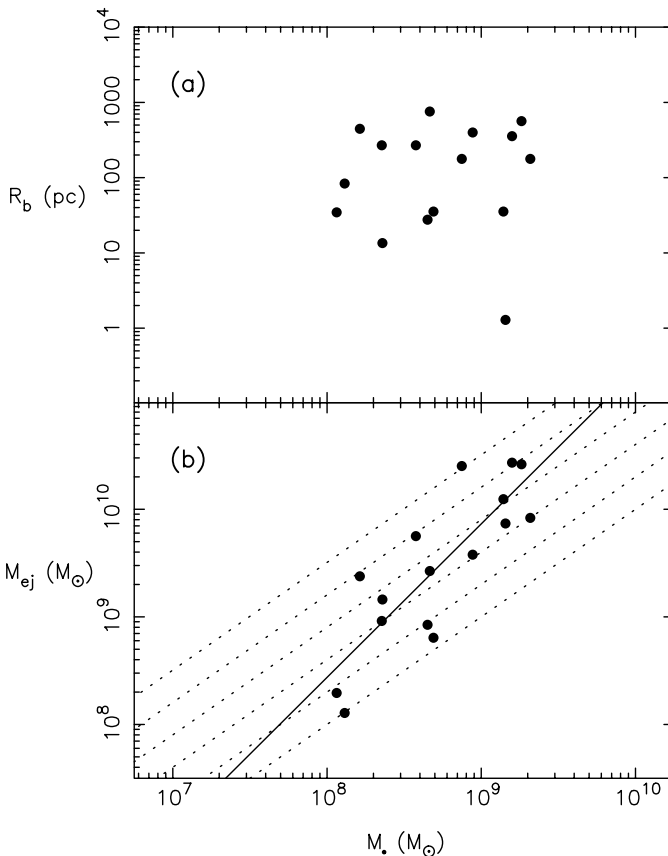


FIG. 19.— Correlation of BH mass with break radius (a) and ejected core mass (b) in galaxies classified as “core” galaxies by Faber et al. (1997). BH masses were computed using the $M_\bullet - \sigma$ relation (equation 30) and measured values of the central velocity dispersion. M_{ej} is defined as in equation (41); the inner slope $\gamma \equiv -d \log \nu / d \log r$ was taken from Gebhardt et al. (1996). Solid line in the lower panel is a least-squares fit. Dotted lines show $M_{ej} = (1, 2, 4, 8, 16, 32)M_\bullet$.

clouds could scatter stars into the BH binary's loss cone (Kim & Morris, 2001), enhance the Brownian motion of the binary, or even perturb the individual BH trajectories before they form a bound pair (Bekki, 2000).

A difficulty with explanations that demand gas accretion is the high accretion rate implied: a mass of order M_{12} must find its way to the nucleus on the time scale of the merger, or roughly one crossing time, so that the binary avoids disrupting the stellar cusp. Much of the energy released from the BH binary would go into heating the gas. It follows that the events we are envisioning are energetically similar to quasars, a conclusion reached also by Graham et al. (1990), Gould & Rix (2000) and others. If this picture is correct, power-law galaxies acquired most of their mass before and during the quasar epoch, from progenitors that were gas-rich, while core galaxies are ellipticals whose most recent major merger occurred after the era of formation of the BHs.

There is circumstantial evidence in support of this picture. The division between core and power-law galaxies occurs at $-22 \lesssim M_V \lesssim -20.5$ (Faber et al., 1997). In semi-analytic models for galaxy formation, the predicted ratio of gas mass to stellar mass during the last major merger is a steep function of galaxy luminosity, dropping from ~ 3 for $M_V = -18$ to ~ 0.3 for $M_V = -21$ (Kauffmann & Haehnelt, 2000). Thus the gas content of the progenitors of the power-law galaxies was likely to have been high at all previous stages in the merger hierarchy, while for core galaxies the most recent mergers were probably gas-poor. Furthermore the redshift of the last major merger (defined as a merger with mass ratio less extreme than 1 : 3) is a strong function of a galaxy's current mass. Most galaxies with masses less than $\sim 10^{10} M_\odot$ ($M_V \approx -18$) have never experienced a major merger; only galaxies with $M \gtrsim 10^{11} M_\odot$ ($M_V \approx -21$) have typically undergone a major merger since a redshift of 1 (Kauffmann, Charlot & Balogh, 2001).

Sharpening these arguments will require N -body simulations of BH binary evolution including gas.

6.5. Coalescence Time Scales and Persistence of Binary BHs

We identify two characteristic time scales associated with the decay of the BH binary in our simulations. The first, which is essentially N -independent, is the brief period following the merger when the two BHs fall to the center and form a hard binary. As discussed above (§3), the BHs initially come together in a time that is approximately as long as the merger itself; most of the energy transfer from the BHs to the stars in the nucleus takes place in just $\Delta t \approx 0.1$ in model units (Figure 4), or $\sim 10^5 - 10^6$ yr. The separation between the BHs at the end of this period, $t \approx t_h$, is $\sim 10^{-3}$ in model units (Figure 1) corresponding to roughly 0.05 pc (M32) or 3 pc (M87).

The second time scale is associated with the gradual decay of the BH binary. We especially wish to know how long it will take the binary to decay to the point that emission of gravitational radiation becomes the dominant energy sink. This occurs when $|\dot{a}/a|^{-1}$ due to mass ejection first equals t_{gr} as defined above (equation 22). This second time scale is potentially N -dependent, since $|\dot{a}/a|$ depends to some extent on collisional processes which may be much larger in our simulations than in real galaxies (§4).

For the moment, assume that the decay rate of the BH binary in our simulations is characteristic of real galaxies with much larger N . The inverse semimajor axis increases roughly linearly with time in our runs (Figure 8b), which suggests that we define a decay rate

$$S \equiv \frac{d}{dt} \left(\frac{1}{a} \right). \quad (42)$$

In model units, $S \approx 7.0 \times 10^2$; in physical units,

$$S \approx 1.0 \times 10^{-6} \left(\frac{M_\bullet}{10^8 M_\odot} \right)^{-0.95} \text{yr}^{-1} \text{pc}^{-1}. \quad (43)$$

We explore the consequences of assuming that S remains constant at this value until gravity-wave coalescence takes place, then discuss the reasonableness of this assumption.

Energy loss due to gravitational radiation dominates that from stellar interactions when

$$t_{gr} = \frac{1}{aS} \quad (44)$$

with t_{gr} given by equation (22), or

$$a^5 = \frac{64}{5S} \frac{G^3 M_{12}^3}{c^5} \quad (45)$$

and $F(e)$ (equation 23) is henceforth set to one. (The modest rates of growth of e in our simulations, §4, imply $F \approx 1$ at all but the latest stages of the merger, and $a \propto F^{-0.2}$.) Combined with equation (43), this condition becomes

$$a < a_{crit} \approx 0.012 \text{ pc} \left(\frac{M_\bullet}{10^8 M_\odot} \right)^{0.8}. \quad (46)$$

Scaling to real galaxies we find $a_{crit} \approx 8 \times 10^{-4}$ pc (M32) and ~ 0.2 pc (M87). These values are smaller, by respective factors of ~ 0.12 and ~ 0.6 , than the final value of a in our simulations. The gravitational radiation time scale when $a = a_{crit}$, which is also approximately equal to the time elapsed in reaching a_{crit} from a_h in this simple model, is

$$t_{gr}(a_{crit}) \approx 5.0 \times 10^7 \text{ yr} \left(\frac{M_\bullet}{10^8 M_\odot} \right)^{0.16} \quad (47)$$

which is 3.0×10^7 yr (M32) and 9.0×10^7 yr (M87). These are factors of ~ 8.0 (M32) and ~ 1.8 (M87) longer than the elapsed time from $t = t_h$ until the final time step in our simulations. Thus a straightforward extrapolation of our N -body results implies that BH binaries would achieve gravitational radiation coalescence in a relatively short time, of order 10^8 yr, following a merger.

Is this a reasonable conclusion? Although the decay rate of the BH binaries in our simulations showed no appreciable N -dependence over the range of N that we tested (Figure 8b), other aspects of the evolution were observed to depend strongly on N , including the amplitude of the Brownian wandering (Figure 9). We argued above (§4) that efficient decay of the binary in our simulations depended on this wandering, since it allowed the binary to interact with a larger pool of stars than if it remained precisely fixed at the center. We predicted that the supply of

stars would have been exhausted by the end of our simulations if the wandering had been reduced by increasing N to $\sim 5 \times 10^5$, still much smaller than in real galaxies.

In a real galactic nucleus, the random velocity of a supermassive BH is expected to be only

$$v_w \equiv \sqrt{\langle v^2 \rangle} \approx 0.033 \text{ km s}^{-1} \left(\frac{m_*}{M_\odot} \right)^{1/2} \left(\frac{M_\bullet}{10^8 M_\odot} \right)^{-0.29} \quad (48)$$

the $M_\bullet - \sigma$ relation has been used to express σ_* in terms of M_\bullet . (This velocity would be increased by a modest factor, of order unity, for a binary BH with $M_{12} = M_\bullet$ (Merritt (2001); Figure 10).) The corresponding wandering radius is

$$r_w \approx 0.010 \text{ pc} \left(\frac{M_\bullet}{10^8 M_\odot} \right)^{-2.35} \left(\frac{r_c}{100 \text{ pc}} \right) \quad (49)$$

with r_c the core radius inside of which the stellar density is taken to be constant. These relations are based on a simple model (Merritt, 2001) in which the stellar distribution is assumed to be unaffected by the motion of the BH or BH binary. If we accept these figures, the wandering radius of a BH binary in a real galaxy is of order or less than the semimajor axis of the binary. It follows that Brownian motion can generally be neglected when considering the interaction of the binary with surrounding stars.

This conclusion may be too pessimistic. A nearly stationary binary would soon “scour clean” a nearly-spherical region of radius $\sim 2a - 3a$ (Zier, 2000), after which the force acting on it would be essentially zero and even a very small v_w would translate into a large displacement. The amplitude of the Brownian motion might therefore slowly increase with time as the binary eats its way through the nucleus. Our simulations tell us nothing about the plausibility of this scenario since our runs never reached the point of loss-cone depletion. Larger N , longer runs, or initial conditions with lower central densities would be required to test this idea.

But suppose that the Brownian motion remains always small, with an amplitude less than the separation between the BHs. We can conservatively estimate the final hardness achieved by the binary by the following argument. Assume that the center of mass of the binary remains fixed, and that the shrinking binary ejects stars in order of their pericenter, from smallest to largest. In this way, the binary acts to reduce the central stellar density in the most rapid possible way, causing its decay to stall in the minimum time. This process would create a hole at the galaxy’s center which grows as the binary shrinks; at some point the binary lies entirely within the hole and its decay ceases. Using Figure 12, it is easy to show that the critical separation is \sim a few times 10^{-4} (assuming that the decay stalls when the hole grows to a radius of a few times a). Scaling to M32, this separation is $a \approx 0.02 \text{ pc}$, and to M87, $a \approx 1 \text{ pc}$. The gravitational radiation time scale at these separations would be 10^{13} yr (M32) and 10^{11} yr (M87), much longer than the age of the universe. In reality, much of the ejected mass in the early stages of the decay comes from stars with larger pericenters, allowing the binary to shrink more than this, say by a factor of ~ 2 . (This is, coincidentally, roughly the final separation reached in our

simulations.) However the basic conclusion is unchanged. In galaxies with initially shallower cusps the decay would be expected to stall at still larger separations.

The inefficiency of stellar-dynamical processes at bringing together supermassive BHs has been noted by a number of authors (e.g. Polnarev & Rees (1994); Valtonen (1996); Merritt (2000); Gould & Rix (2000)). We believe that the problem has often been overstated due to the use of over-simplified models for describing the interaction of stars with the binary; for instance, Gould & Rix (2000) ignore the fact that most of the hardening comes from stars on orbits with apocenters much greater than the semimajor axis a . But the basic argument is sound: a *fixed* binary would have difficulty interacting with several times its own mass in stars even if located at the center of a steep density cusp, hence it could probably not achieve gravitational radiation coalescence in a time shorter than the age of the universe. Unless additional mechanisms exist for extracting energy, its decay would be expected to stall.

We discussed above what these “additional mechanisms” might be and argued that they would be most effective in galaxies whose progenitors were gas-rich. These galaxies, which in our opinion are *least* likely to contain BH binaries, are also the systems in which detection of BH binaries would be easiest, through the measurement of periodically varying features in the emission line systems associated with one or both of the BHs (Begelman, Blandford & Rees, 1980; Gaskell, 1995). Such features have tentatively been detected in a few active galaxies; the best case is probably OJ 287, a blazar with nearly-periodic outbursts dating back roughly 100 years (Pursimo et al., 2000). However the interpretation in terms of a binary system (Lehto & Valtonen, 1996) is not airtight. A larger number of interacting systems exhibit emission from two resolved peaks, probably the nuclei of galaxies in the early stages of merging (e.g. Carico et al. (1990)); the smallest projected separation, in ARP 220, is $\sim 360 \text{ pc}$ (Scoville et al., 1998). A recently-discovered double quasar contains two peaks with a projected separation of a few kiloparsecs (Junkkarinen et al., 2001).

The supermassive BHs in these interacting systems have not yet formed bound pairs. True BH binaries – at separations $a \lesssim a_h$ (equation 4) – would most likely be found in galaxies with low central densities and little gas. Recent formation via mergers, and a high ongoing accretion rate (assuming that the accreted galaxies also contain BHs), would also be propitious. These characteristics constitute almost a textbook definition of a cD galaxy, particularly a multiple-nucleus cD in a rich galaxy cluster (e.g. Schneider & Gunn (1982)). Most cD galaxies are too distant for single, much less double, BHs to be detected kinematically, although a strong case can be made for dual BHs in 3C75, the central radio source in A400. This galaxy exhibits a pair of radio jets that appear to be emitted from point sources separated by $\sim 7 \text{ kpc}$ in projection (Owen et al., 1985). But a more likely separation for a binary BH would be the much smaller distance at which the decay is expected to stall, $0.01 \text{ pc} \lesssim a \lesssim 1 \text{ pc}$. Binaries with these separations might barely be detectable in nearby galaxies by using VLBI techniques to resolve the compact radio sources associated with the individual BHs (Slee et al., 1994).

The existence of uncoalesced BH binaries in galactic nuclei has a number of interesting consequences. BH ejections would result whenever a third supermassive BH, or a second BH binary, is brought into the nucleus following a merger (Valtonen, 1996). In a massive galaxy like a cD, a quasi-steady state might be set up in which the ejection of BHs from the nucleus is matched by the infall of new BHs from the “multiple nuclei” and from previous ejections. If BH ejections are common, most supermassive BHs might be located far from the centers of galaxies, and the mean mass density of BHs in the universe could be much greater than the mass density inferred from nuclear kinematical studies (Merritt & Ferrarese, 2001b).

6.6. Centers of Dark-Matter Halos

Cold dark matter (CDM) simulations of the growth of structure in the universe (Moore et al., 1998; Jing, 2000; Bullock et al., 2001) predict dark-matter halos with steep central density cusps, $\rho \sim r^{-\gamma}$, $1 \lesssim \gamma \lesssim 2$, similar to the cusps in our initial models. The dense (baryonic) regions in which supermassive BHs first formed were probably located near the centers of these halos (Haehnelt, Natarajan & Rees, 1998). Cosmological simulations currently lack the resolution to handle compact massive objects like BHs, but a number of authors (Ipser & Sikivie, 1987; Gondolo & Silk, 1999; Ullio, Zhao & Kamionkowski, 2001) have investigated the response of pre-existing dark matter halos to the growth of supermassive BHs. An initially steep dark-matter cusp becomes even steeper within the radius of influence r_{gr} of the BH, $\rho_{DM} \sim r^{-A}$, $2.25 \leq A \leq 2.5$ (Gondolo & Silk, 1999). This result has been claimed to be inconsistent with experimental upper bounds on annihilation radiation from the Galactic center (Gondolo & Silk, 1999; Bertone, Silk & Sigl, 2000; Gondolo, 2000), implying either that dark matter cusps do not exist, or that current ideas about the composition of the dark matter are wrong.

If mergers of dark-matter halos occurred after the BHs were in place, however, the effect of the BHs on the dark matter density would be roughly the opposite of what these studies assume: the BHs would tend to destroy the cusps via ejection of dark matter particles. Just the first step in the BH merger process – formation of a hard binary via dynamical friction – transfers enough energy to the background to convert an r^{-2} cusp into a shallower, $\sim r^{-1}$ cusp within a radius that contains several times the BHs’ mass (§3; Figure 7). (This conclusion is independent of any uncertainties about Brownian motion, loss-cone refilling, etc., which would be negligible anyway in a dark-matter-dominated cusp.) Most of the annihilation radiation from a putative dark-matter cusp would come from a region smaller than this; reducing the cusp slope from ~ -2 to ~ -1 lowers the predicted flux by several orders of magnitude (Gondolo & Silk (1999), Fig. 2). The fact that steep cusps persist in the *stellar* density in many galaxies, including the Milky Way (Alexander, 1999), suggests that dark matter cusps might also sometimes avoid destruction. But the mechanisms discussed above for preserving steep cusps in the stellar distribution – e.g. star formation from infalling gas – are less applicable to dark matter. Furthermore we expect the coupling between baryons and dark matter to be less than perfect, and stellar cusps would themselves inject energy into the dark matter as they spiralled to the center of the merging dark matter halos; a

similar effect is seen in the merger simulations of Merritt & Cruz (2001). Thus it seems possible that dark matter cusps could be destroyed even in galaxies which manage to retain steep cusps in the stellar distribution.

Could the mechanisms discussed in this paper be relevant to the low apparent density of dark matter at the centers of dwarf and low-surface-brightness galaxies (Flores & Primack, 1994; de Blok & McGaugh, 1997; McGaugh & de Blok, 1998; de Blok et al., 2001)? The density increases only as $\rho_{DM} \sim r^{-0.2}$ at the centers of these galaxies (de Blok et al., 2001), much flatter than predicted by CDM models. It is unlikely that BHs alone are responsible for this deficit, however, for several reasons. There is currently no evidence for supermassive BHs in these galaxies, and scaling relations like the $M_\bullet - \sigma$ relation would suggest small values of M_\bullet . The inferred core radii of the dark matter halos are very large, of order $10^2 - 10^3$ pc, implying an ejected mass that is much greater than any likely value of M_\bullet . Dwarf and low-surface-brightness galaxies are also unlikely to have had active merger histories, at least since the era of formation of the BHs. The only possibility we can see for destruction of dark-matter cusps on the observed scales in these galaxies would be the existence of a significant population of condensed objects that pre-date the dark matter halos, such as primordial BHs (Carr, 1985).

7. CONCLUSIONS

1. Mergers of equal-mass stellar systems containing supermassive black holes and steep central density cusps produce nuclei with shallow cusps, $\rho \sim r^{-1}$, inside of a break radius r_b , similar to the luminosity profiles observed at the centers of bright elliptical galaxies. Most of the evolution in the central density occurs within a short time, $\sim 10^6 - 10^7$ yr, after the black holes form a binary; the cusp continues to flatten thereafter as the binary ejects stars via the gravitational slingshot. The dependence of core properties on black hole mass in observed galaxies is shown to be consistent with this formation model.

2. The merger-induced rotation in the nucleus is reduced significantly by the binary as it preferentially ejects stars whose angular momenta are aligned with its own. The stellar velocity dispersion tensor in the nucleus becomes mildly tangentially anisotropic as well, although this effect is too small to be easily observed in real galaxies.

3. Hardening of the black-hole binary takes place efficiently in our simulations due to the large supply of stars provided by the dense cusps, and also to the Brownian motion of the binary, which allows it to interact with a larger number of stars than if it remained fixed. There is no significant dependence of the binary hardening rate on number of particles N and the binary’s loss cone never approaches depletion, in spite of the fact that the decay is followed over a factor of ~ 20 in semimajor axis after it first forms a hard binary, considerably farther than in earlier simulations. The hardening rate that we measure, if it remained constant, would imply gravitational-radiation coalescence in a relatively short time, of order 10^8 years, following the merger.

4. However, we argue that the decay of a black-hole binary in a real galactic nucleus would sometimes be expected to stall at separations of $0.01 - 1$ parsec due to depletion of the stellar loss cone around a nearly-stationary

binary. At these separations, gravitational radiation would be ineffective at inducing coalescence and the binary would persist indefinitely, unless some other physical process were able to extract its binding energy. We argue that uncoalesced black-hole binaries are most likely to be found in the nuclei of cD or other giant elliptical galaxies.

5. If we artificially combine the two black holes just after they form a hard binary, the merger remnant preserves its steep, $\rho \sim r^{-2}$ density cusp. We propose this as a model for the retention of steep cusps in the “power-law” galaxies, and suggest that these galaxies experienced their last major mergers during the quasar epoch.

6. Our simulations can also be interpreted as describing mergers of dark-matter cusps containing supermassive black holes. We argue that the steep cusps predicted by cold-dark-matter cosmologies would be destroyed by binary black holes in galaxies where the stellar cusps are also destroyed.

This work was supported by NSF grants AST 96-17088 and 00-71099 and by NASA grants NAG5-6037 and NAG5-9046. We thank Sverre Aarseth, Marc Hemsendorf and Rainer Spurzem for their patient and expert guidance with the N -body codes **NBODY6** and **NBODY6++**, and for making their programs available to us in advance of general release. Without their generous help this project would not have been possible. We discussed the potential observability of binary supermassive black holes with E. Sadler and J. Wrobel. The pre-hard-binary merger simulations presented here (§2) were carried out by Fidel Cruz using the tree code **GADGET** on the Rutgers HPC-10000 supercomputer; we are indebted to him and to Volker Springel for advice about using this code. Dr. Cruz was supported by a fellowship from the Consejo Nacional de Ciencia y Tecnología de México. This work was partially supported by the National Computational Science Alliance under grant no. MCA00N010N and utilized the San Diego Supercomputer Center Cray T3E. We are also grateful to the Center for Advanced Information Processing at Rutgers University for their generous allocation of computer time on the HPC-10000.

APPENDIX A

THE COULOMB LOGARITHM

Estimates of the orbital decay rate due to dynamical friction in §3 were dependent on the Coulomb logarithm $\ln \Lambda$. Here we derive estimates for $\ln \Lambda$ in the two cases of interest: a single massive object near the center of a stellar system with a steep density profile; and a sphere of finite size representing a merging cusp. In both cases we find $\ln \Lambda \approx 1$.

The deceleration of a massive test particle due to dynamical friction is often written in the form (e.g. Chandrasekhar (1943); Spitzer (1987))

$$\langle \Delta v_{\parallel} \rangle = -\frac{4\pi G^2 M \rho \ln \Lambda F(v)}{v^2} \quad (\text{A1})$$

where M and v are the mass and the velocity of the test particle, ρ is the density of light field particles, $F(v)$ is the fraction of field particles with velocities less than v , and $\ln \Lambda$ is the Coulomb logarithm that arises in the integration over a field of constant density. The force exerted on the test particle by an infinite homogeneous field diverges and a cutoff in the form of a maximum impact parameter p_{max} is required. The dependence of $\langle \Delta v_{\parallel} \rangle$ on p_{max} is logarithmic:

$$\langle \Delta v_{\parallel} \rangle \propto \ln \sqrt{1 + \frac{p_{max}^2}{p_{min}^2}} \equiv \ln \Lambda \quad (\text{A2})$$

where p_{min} is a minimum impact parameter cutoff that also needs to be specified in context. In treatments where the field particles are assumed to move along rectilinear orbits (e.g. Chandrasekhar & von Neumann (1942)), the integral leading to (A2) diverges at low impact parameters. The divergence vanishes when the proper Keplerian trajectories are used; field stars of a given relative velocity V_0 then transmit a net momentum proportional to $\ln \sqrt{1 + p_{max}^2/p_0^2}$ with $p_0 = GM/V_0^2$, and integration over V_0 gives a p_{min} in equation (A2) of order GM/σ^2 with σ the 1D velocity dispersion of the field stars. For instance, when the velocity v of the test star is much less than σ , $p_{min} \approx GM/\sqrt{2}\sigma^2$, roughly the radius of gravitational influence r_G of the massive object (Merritt, 2001).

A variety of prescriptions can be found in the literature regarding the optimal and most accurate choice for p_{max} , and thus for $\ln \Lambda$. Since all gravitationally-bound structures in the universe have finite extent, the Coulomb logarithm is in theory a real physical quantity subject to calculation if one is ready to abandon several simplifying assumptions that enter equation (A1). In practice, the full-fledged phase-space integration is cumbersome at best and numerical N -body treatments often resort to the fitting of equation (A1) to the dynamical drag observed in simulations.

Maoz (1993) derived an expression for dynamical friction in an inhomogeneous isothermal Maxwellian background which reads

$$\langle \Delta v_{\parallel} \rangle = -\frac{\sqrt{2}G^2 M}{v\sigma} \int d^3r \frac{\rho(\mathbf{r})\alpha}{|\mathbf{R} - \mathbf{r}|^3} \left[e^{\alpha^2 - x^2} \text{erf}(\alpha) - 1 \right] \Theta(|\mathbf{R} - \mathbf{r}| - d) \quad (\text{A3})$$

where $\alpha \equiv \mathbf{x} \cdot (\mathbf{R} - \mathbf{r})/|\mathbf{R} - \mathbf{r}|$ and $\mathbf{x} \equiv \mathbf{v}/\sqrt{2}\sigma$, while \mathbf{R} is the position of the test particle and $\Theta(y) = 1$ when $y > 0$ and is zero otherwise. The Θ -function serves to exclude a finite small volume of radius d around the particle from integration, necessary since in Maoz's treatment the field-star trajectories are assumed to be straight lines.

If the stellar system is spherical and centered on the test particle, the radial and the angular integrals in equation (A3) can be separated

$$\langle \Delta v_{\parallel} \rangle = -\frac{\sqrt{2}G^2 M}{v\sigma} \left(\frac{2\pi}{x} \right) \left\{ e^{-x^2} \int_{-x}^x \alpha e^{\alpha^2} \text{erf}(\alpha) d\alpha \right\} \left\{ \int_d^\infty \frac{\rho(r)}{r} dr \right\}. \quad (\text{A4})$$

The first factor in braces can be identified with the velocity factor $F(v)$ appearing in equation (A1)

$$e^{-x^2} \int_{-x}^x \alpha e^{\alpha^2} \text{erf}(\alpha) d\alpha = \text{erf}(x) - x \text{erf}'(x) \equiv F(v) \quad (\text{A5})$$

The second factor in braces encapsulates the dependence of dynamical friction on the radial distribution of field particles. The formula becomes

$$\langle \Delta v_{\parallel} \rangle = -\frac{4\pi G^2 M F(v)}{v^2} \int_d^\infty \frac{\rho(r)}{r} dr \quad (\text{A6})$$

which can be compared with equation (A1) to arrive at a definition of the Coulomb logarithm in terms of an arbitrarily chosen fiducial density ρ :

$$\rho \ln \Lambda \equiv \int_d^\infty \frac{\rho(r)}{r} dr. \quad (\text{A7})$$

Clearly, Maoz's formula reduces to equation (A2) if the density is constant in an annulus with inner and outer radii d and p_{max} and vanishes outside.

Real stellar systems do not have large-radius density cutoffs but the density typically decays as a power law $\rho \sim r^{-\lambda}$. The spatial integral in equation (A3) converges for *any* $\lambda > 0$, and one is free to take the limit $p_{max} \rightarrow \infty$. To illustrate

this, we calculate the contribution due to the far field ($r \geq d$) particles obeying the simplest power-law profile centered on the test particle:

$$\rho(r) = \rho_0 \left(\frac{r}{d}\right)^{-\lambda}. \quad (\text{A8})$$

Assume for the moment that $\rho(r) = 0$ when $r < d$, reflecting a “hole” in the density cusp due to, e.g., loss-cone depletion around a compact massive object. In this case the Coulomb logarithm must be chosen as

$$\rho \ln \Lambda = \rho_0 \int_d^\infty \left(\frac{r}{d}\right)^{-\lambda} \frac{dr}{r} = \rho_0 \frac{1}{\lambda} = \frac{\rho(d)}{\lambda} \quad (\text{A9})$$

leading to dynamical friction that is proportional to the density at the hole’s inner edge and inversely proportional to the logarithmic slope:

$$\langle \Delta v_{\parallel} \rangle = -\frac{4\pi G^2 M F(v) \rho(d)}{\lambda v^2}. \quad (\text{A10})$$

This result helps expose the inadequacy of conclusions drawn in the context of homogeneous backgrounds where greatest contribution to the drag force comes from distant encounters ($r \gg d$). In our case, the fractional contribution from near-field particles at distances $d \leq r \leq 2d$ amounts to $1 - 2^{-\lambda}$ which is more than 50% when $\lambda > 1$. Also, while in view of the traditional choice $\ln \Lambda = p_{\max}/p_{\min}$ one is prone to expect $p_{\min} \ll p_{\max}$, we find that $p_{\max}/p_{\min} \sim e^{1/\lambda} \sim 1$ and any meaningful choice for the effective p_{\max} would reflect neither Chandrasekhar’s “average distance between stars” nor the “size of the system.”

In the absence of a hole of radius d around the test particle, the integral in (A7) may still diverge. This divergence is an artefact of an approximation employed by Maoz whereby stars move along straight lines; it vanishes if exact Keplerian trajectories are computed for the field stars, in which case the effective d is roughly $r_G = GM/\sigma^2$ (Spitzer, 1987). In applying equation (A10) to the orbital decay of a single BH near the center of a stellar system in §3, we take $\ln \Lambda = 1/2$ corresponding to $\gamma = 2$ and use for $\rho(d)$ the mean density within a radius of 0.01, roughly the radius of gravitational influence of the BH.

The second context in which we applied the dynamical friction formula in §3 was the orbital decay of two finite-density spheres representing the original cusps of the merging stellar systems. We model such a test object with a spherical mass distribution $M(r)$ and assume that it is much denser than the field environment. According to Gauss’ theorem, field particles coming to within pericenter distance p from the center of the test object do not interact with the entire object but only with a portion of mass $M(p)$. This suggests an immediate modification of equation (A3):

$$\langle \Delta v_{\parallel} \rangle = -\frac{\sqrt{2}G^2}{v\sigma} \int d^3r \frac{M(|\mathbf{R} - \mathbf{r}|) \rho(\mathbf{r}) \alpha}{|\mathbf{R} - \mathbf{r}|^3} \left[e^{\alpha^2 - x^2} \text{erf}(\alpha) - 1 \right] \Theta(|\mathbf{R} - \mathbf{r}| - d). \quad (\text{A11})$$

As an application consider estimating the orbital decay rate for a pair of overlapping Jaffe model galaxies in the final stages of a merger proceeding along a circular orbit. Each galaxy has density

$$\rho(r) = \frac{M}{4\pi r_0^3} \left(\frac{r}{r_0}\right)^{-2} \left(1 + \frac{r}{r_0}\right)^{-2} \quad (\text{A12})$$

and thus $M(r) = Mr/r_0$ and $\sigma \approx (GM/2r_0)^{1/2}$. Speed of the test body relative to the other body is twice the circular velocity $v = 2v_c = 2(2^{-1/2}\sigma)$, while α can be expressed terms of the angle θ between \mathbf{r} and \mathbf{R} ,

$$\alpha = -\frac{x r \sin \theta}{|\mathbf{R} - \mathbf{r}|}. \quad (\text{A13})$$

Inspection reveals that now, with the test body spread out in space, the limit $d \rightarrow 0$ can be taken inside the integral, hence the Θ -function is identical to unity.

With these substitutions equation (A11) can be integrated numerically. We emphasize that treating the test galaxy as a rigid body is a crude approximation; in reality there will be an outer tidal radius beyond which the galaxies are indistinguishable. The integral depends strongly on the choice of tidal radius. A natural choice is the separation between the centers $R = a$, which yields

$$\langle \Delta v_{\parallel} \rangle \approx -\frac{1.50\sigma^2}{a}. \quad (\text{A14})$$

APPENDIX B

COMPUTING DENSITY PROFILES

Here we present the algorithms which we used to derive smooth estimates, $\hat{\nu}(r)$ and $\hat{\Sigma}(R)$, of the particle number density and surface density profiles from the N -body positions.

The routines in **MAPEL** (Merritt, 1994) allow one to derive maximally unbiased estimates of ν and Σ using penalty functions that embody the approximate power-law nature of these functions. However the **MAPEL** routines are relatively slow, and this fact presented difficulties when constructing estimates using the $N \sim 10^6$ particle data sets consisting of superposed N -body output at several time steps. Kernel based algorithms are faster but potentially more biased; however we found them to be adequate for all but the most steeply rising ($\nu \sim r^{-2}$) profiles and so adopted them here.

Our derivation follows that in Merritt & Tremblay (1994). In the absence of any symmetries in the particle distribution, a valid estimate of the number density ν corresponding to a set of particle positions \mathbf{r}_i is

$$\hat{\nu}(\mathbf{r}) = \sum_{i=1}^N \frac{1}{h^3} K \left[\frac{1}{h} |\mathbf{r} - \mathbf{r}_i| \right] \quad (\text{B1})$$

where h is the window width and K is a normalized kernel, e.g. the Gaussian kernel:

$$K(y) = \frac{1}{(2\pi)^{3/2}} e^{-y^2/2}. \quad (\text{B2})$$

Now imagine that each particle is smeared uniformly around the surface of the sphere whose radius is r_i ; typically this sphere will be centered on the single or binary BH. If the density profile is actually spherically symmetric, this smearing will leave the density unchanged; if not, it will produce a spherically symmetric approximation to the true profile. The spherically-symmetrized density estimate is

$$\hat{\nu}(r) = \sum_{i=1}^N \frac{1}{h^3} \frac{1}{4\pi} \int d\phi \int d\theta \sin \theta K \left(\frac{d}{h} \right), \quad (\text{B3a})$$

$$d^2 = |\mathbf{r} - \mathbf{r}_i|^2 \quad (\text{B3b})$$

$$= r_i^2 + r^2 - 2rr_i \cos \theta \quad (\text{B3c})$$

where θ is defined (arbitrarily) from the \mathbf{r}_i -axis. This may be written in terms of the angle-averaged kernel \tilde{K} :

$$\hat{\nu}(r) = \sum_{i=1}^N \frac{1}{h^3} \tilde{K}(r, r_i, h), \quad (\text{B4a})$$

$$\tilde{K}(r, r_i, h) \equiv \frac{1}{4\pi} \int d\phi \int d\theta \sin \theta K \left(h^{-1} \sqrt{r_i^2 + r^2 - 2rr_i \cos \theta} \right) \quad (\text{B4b})$$

$$= \frac{1}{2} \int_{-1}^1 d\mu K \left(h^{-1} \sqrt{r_i^2 + r^2 - 2rr_i \mu} \right). \quad (\text{B4c})$$

Substituting for the Gaussian kernel, we find

$$\tilde{K}(r, r_i, h) = \frac{1}{(2\pi)^{3/2}} \left(\frac{rr_i}{h^2} \right)^{-1} e^{-(r_i^2 + r^2)/2h^2} \sinh(rr_i/h^2). \quad (\text{B5})$$

A better form for numerical computation is

$$\tilde{K}(r, r_i, h) = \frac{1}{2(2\pi)^{3/2}} \left(\frac{rr_i}{h^2} \right)^{-1} \left[e^{-(r_i - r)^2/2h^2} - e^{-(r_i + r)^2/2h^2} \right]. \quad (\text{B6})$$

We want to vary the window width with position in such a way that the bias-to-variance ratio of the estimate is relatively constant. Let h_i be the window width associated with the i th particle. The density estimate based on a variable window width is

$$\hat{\nu}(r) = \sum_{i=1}^N \frac{1}{h_i^3} \tilde{K}(r, r_i, h_i). \quad (\text{B7})$$

The optimal way to vary h_i is according to Abramson's (1982) rule:

$$h_i \propto \nu^{-\alpha}(r_i), \quad \alpha = 1/2. \quad (\text{B8})$$

Since we don't know $\nu(r_i)$ a priori, we compute a pilot estimate of ν using a fixed kernel and adjust the h_i based on this estimate (Silverman, 1986).

The surface density profile could be computed via simple projection of $\hat{\nu}(r)$. Instead, we computed $\hat{\Sigma}(R)$ directly from the coordinates projected along one axis. The two-dimensional kernel estimate of $\Sigma(\mathbf{R})$ in the absence of any symmetries is

$$\hat{\Sigma}(\mathbf{R}) = \sum_{i=1}^N \frac{1}{h^2} K' \left[\frac{1}{h} |\mathbf{R} - \mathbf{R}_i| \right] \quad (\text{B9})$$

where K' is the two-dimensional Gaussian kernel,

$$K'(y) = \frac{1}{2\pi} e^{-y^2/2}. \quad (\text{B10})$$

Now smear each particle uniformly in angle ϕ at fixed R_i . The density estimate becomes

$$\hat{\Sigma}(R) = \sum_{i=1}^N \frac{1}{h^2} \frac{1}{2\pi} \int K' \left(\frac{d}{h} \right) d\phi, \quad (\text{B11a})$$

$$d^2 = R_i^2 + R^2 - 2RR_i \cos \phi. \quad (\text{B11b})$$

In terms of the angle-averaged kernel \tilde{K}' :

$$\hat{\Sigma}(R) = \sum_{i=1}^N \frac{1}{h^2} \tilde{K}'(R, R_i, h), \quad (\text{B12a})$$

$$\tilde{K}'(R, R_i, h) \equiv \frac{1}{2\pi} \int K' \left(h^{-1} \sqrt{R_i^2 + R^2 - 2RR_i \cos \phi} \right) d\phi \quad (\text{B12b})$$

$$= \frac{1}{2\pi} e^{-(R_i^2 + R^2)/2h^2} I_0(RR_i/h^2) \quad (\text{B12c})$$

where the last expression was derived using the Gaussian kernel; I_0 is the modified Bessel function.

APPENDIX C

LOSVD EXTRACTION

We carried out non-parametric recovery of the LOSVDs $N(V)$ on a dense velocity grid, $-V_0 < V < V_0$, by maximizing the penalized log-likelihood (Merritt, 1997)

$$\log \mathcal{L}_P[N, V_i] = \sum_{i=1}^n \log N(V_i) - \alpha P[N] - n \int_{-V_0}^{V_0} N(V) dV \quad (\text{C1})$$

where n is the number of particles inside an aperture, V_i is the line-of-sight projection of the i th particle's velocity, and $P[N]$ is a natural choice for the penalty functional that is large for noisy N but assigns zero penalty to a Gaussian function (Silverman, 1986)

$$P[N] = \int_{-V_0}^{V_0} \left[\left(\frac{d}{dV} \right)^3 \log N(V) \right]^2 dV. \quad (\text{C2})$$

When α is very large, maximization of $\log \mathcal{L}_P$ yields a pure Gaussian distribution. For the purpose of extracting Gauss-Hermite (GH) moments, we chose α smaller than necessary for smooth LOSVDs, thereby ensuring that non-Gaussian substructure of the distributions is not compromised by smoothing.

Once an LOSVD is available, parameters in the GH expansion can be readily calculated. Define the GH moments of N as

$$h_i[N] = 2\sqrt{\pi} \int_{-\infty}^{\infty} N(V) g(w) H_i(w) dV \quad (\text{C3})$$

where H_i are the Hermite polynomials as defined by Gerhard (1993) and the weight function

$$g(w) = \frac{1}{\sqrt{2\pi}\gamma_0} e^{-w^2/2}, \quad w = \frac{V - V_0}{\sigma_0} \quad (\text{C4})$$

has three free parameters $(\gamma_0, V_0, \sigma_0)$. Following van der Marel & Franx (1993), we choose these parameters such that $h_0[N] = 1$ and $h_1[N] = h_2[N] = 0$. This can be achieved by minimizing the sum $(h_0 - 1)^2 + h_1^2 + h_2^2$ as a function of $(\gamma_0, V_0, \sigma_0)$. Once these parameters are uniquely determined, the higher-order GH moments, including h_3 and h_4 , can be evaluated from equation (C3) by numerical integration.

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